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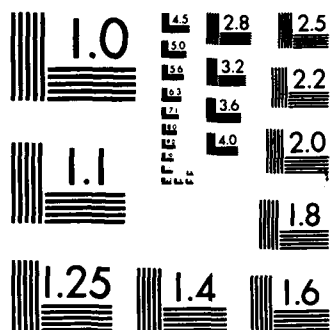
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## THESIS

SENSITIVITY OF  
ALIVENESS ADJUSTMENTS

by

Theodore J. Janosko

June 1987

Thesis Advisor

Donald R. Barr

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Sensitivity of  
Aliveness Adjustments

by

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Submitted in partial fulfillment of the  
requirements for the degree of

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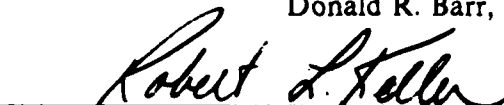
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## ABSTRACT

Aliveness analysis is a computational technique which attempts to estimate the expected number of losses had real ordnance been used during a force-on-force experiment. This thesis carefully follows the development and motivation for the aliveness concept. Examples of aliveness computations are presented, with special emphasis on the SGT York Follow-on-Evaluation (FOE). A simple aliveness computer program was used to examine the sensitivity of aliveness adjustments to changes in such parameters as probability of kill and target selection method.



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## I. INTRODUCTION

Talk about jumping out of the frying pan and into the fire! After sweating blood to complete your thesis in time to graduate from The Naval Postgraduate School, you now find yourself enroute to the United States Army's largest laboratory, Fort Hunter-Liggett, California, just two days after reporting in to your new unit. The U. S. Army has turned to the Combat Developments Experimentation Command (CDEC) to support or kill the largest Air Defense Artillery system in the U. S. Army's history, the Sergeant York project. The primary test-design officer was compassionately reassigned last week and your new supervisor wants you to exploit the skills certified in your recently acquired operations research degree.

The special testing units and the prototype equipment are returning to Fort Bliss, Texas, and all that remains to complete the five million dollar follow-on evaluation (FOE) is to analyze the data and write the report. As you examine the data you uncover some problems. Not all engagements went to resolution because of computer and instrumentation problems, but some of those may be recovered by painstakingly examining the video recordings. You also discover that the probability of kill (Pk) for the opposing force helicopter is wrong; that model of helicopter is no longer the primary rotary aircraft threat. This means that some of the players were killed that should have lived. Had they lived, they may have killed other players, and so on. Should you recommend that all of the equipment and test units be recalled and spend another five million dollars and even more time? What should you do?

Although not supported by the entire Army operations research community, the aliveness concept developed by Dr. Marion R. Bryson of CDEC and Dr. Carl T. Russell of the U. S. Army Operational Test and Evaluation Agency (OTEA) appears to be an answer. It is a means of resolving problems such as these, and it can reduce the variance of experimental results or decrease the number of trials required for the same variance, obviously at a reduction in cost. It is hoped that this thesis will shed some light on the aliveness concepts. It will analyze aliveness through simulation and analyze the sensitivity of the results to changes in the parameters. Chapter Two traces the development of the aliveness concept. Chapter Three presents the results from simulation trials with varying battle parameters such as probability of kill and target

selection method and the analysis of those results. Chapter Four summarizes the results from the simulation trials. Chapter Five offers conclusions about the sensitivity of the aliveness computations to variation in battle parameters and recommendations of areas for further investigation and study.

## II. DEVELOPMENT OF ALIVENESS

### A. HISTORICAL BACKGROUND

As late as 1972, the U. S. Army Combat Developments Experimentation Command (CDEC) was using the total number of casualties from field exercises to estimate the expected casualties [Ref. 1]. CDEC's civilian scientific support organization at that time, Stanford Research Institute (SRI), used the "Global" model to estimate the expected casualties. Many costly repetitions of field exercises were conducted in order to gain statistical stability. Mr. Vince Finn of SRI suggested that one hundred-twenty runs of a particular field experiment were required to obtain the desired statistical stability in estimates obtained from the field experiment.

At this point, Dr. Marion R. Bryson suggested an alternate method to estimate the expected casualties. He suggested that real casualties as determined by the random number draws still be used to "shape the battlefield," but not be used to gather statistics for analytical purposes. Dr. Bryson suggested that the summation of the probability of kill ( $P_k$ ) would be a much better statistical estimator. He demonstrated that, by using the summation of the probability of kills, the number of trials required in experiments decreased significantly: for example, from one hundred-twenty to fifteen for the same variance of an estimator of interest.

Dr. Bryson acknowledges that an effective real time casualty assessment (RTCA) system to shape the battlefield is still needed. The Real Time Casualty Assessment System adjudicates engagements in near real time and assesses casualties, that is, "kills" a player or allows him to survive, each time the player is fired upon. These casualties or kills are needed to force the players to behave in a realistic fashion; however, instead of adding the number of kills to estimate the mean number of casualties, the probabilities of kill during each engagement are added.

### B. REAL TIME CASUALTY ASSESSMENT

The U. S. Army Combat Developments Experimentation Command's primary mission is to test equipment, tactics, and doctrine in a realistic environment. The analysts at CDEC believe that the most effective way of measuring the performance of combat systems in a realistic environment is to stage simulated battles in which real soldiers manning real weapon systems oppose each other on real terrain. Real Time

Casualty Assessment (RTCA) experimentation attempts to model, in the field laboratory, the actual battlefield conditions including real time attrition of the friendly and opposing forces based on engagements that occur during their respective tactical actions. Attrition on the actual battlefield is a result of physical damage to a weapon system or its crew. In a Real Time Casualty Assessment experiment, a central computer attrits each force by neutralizing the firing mechanism of the weapon system or by sending the player a message ordering him out of play. This procedure is accomplished in near real time, usually within three seconds of the target player having been "paired" by a simulated round firing of an opposing force weapon system and being assessed a casualty by the computer. [Ref. 2]

A typical experiment conducted by the Combat Developments Experimentation Command (CDEC) consists of small units, usually no larger than battalion-level, and more often involving company-level or smaller forces. The experiment is often a two-sided free play exercise with each side having conflicting goals or objectives. The Blue side represents friendly forces and the Red side represents opposing forces using weapon systems and tactics of a postulated enemy. The weapon systems used by the forces are often surrogates. For example, the opposing force players use a comparable weapon for the experiment, with the correct opposing force weapon characteristics (rate of fire, effective range, probabilities of kill, etc.) loaded into the computer. The trials are highly instrumented, which is one of the limiting constraints on the size of the forces involved. Casualties are assessed by the computer and removed in near real time. The battles are of a short duration, usually between twenty and sixty minutes, until one of the forces achieves its objective or both forces are non-mission capable.

Since many of the CDEC-conducted experiments are free-play, the players automatically perform many of the steps of the multi-step ordered direct fire engagement process. By their actions the players ensure that line-of-sight between the firer and target exists, that there is a detection, and that there is a decision to fire. The final steps of the process, the firing and assessment steps, are usually conducted by the computer. To simulate the weapon system interactions, each player is equipped with a sophisticated electronic instrumentation package. The instrumentation package used in the experiment usually consists of a laser to simulate the firing of the weapon system, laser detectors to record a hit, and a position locating system to measure the range and relative geometry. CDEC is also looking to perfect a microwave instrumentation system known as Engagement Line-of-Sight System (ELOSS). The two components of

ELOSS are omni-directional emitting interrogators and transponders. Since microwave requires visual line-of-sight for transmission, only those transponders that have intervisibility with an interrogator will receive the signal. The transponders which receive a signal, echo the signal back to the sending interrogator and the sending interrogator immediately records the existence of line-of-sight with that transponder. Only one pair of interrogators and transponders are on the air at one time, making it possible to determine which pair of players are intervisible [Ref. 3]. The instrumentation package may also include voice recording systems, cameras, firing signature simulators, and attitude heading reference systems.

The last two steps of an direct fire engagement consist of the following actions. The firer, firing blank ammunition or activating a firing signature simulator, activates a laser which is boresighted with the weapon. Simultaneously with the activation of the laser, a firing message is sent to the computer. If the firer's aim is accurate, the laser beam is detected by the laser sensors on the target and a hit message is sent to the computer. From telemetry, through ground stations, and transponders and interrogators on weapon platforms, the computer calculates the engagement range. The computer looks up the probability of kill ( $P_k$ ) from predetermined tables for this firer-target pair at the given range. A uniform (0,1) random number is drawn and compared to the  $P_k$  and the outcome of the engagement is determined. If the target is assessed by the computer to be a casualty, the target weapon system's firing mechanism is neutralized and the target is notified by the computer. The crew of the target weapon system ceases all tactical actions and releases a smoke cue to inform other players of their casualty status. [Ref. 4]

Obviously if there are several different weapon systems, the probability of kill ( $P_k$ ) tables can get very extensive. The typical  $P_k$  tables list the probability of kill for a specific weapon system pair at various ranges. These range increments can vary from ten to five hundred meters based on what the test-design officer requested from the U. S. Army Materiel Systems Analysis Activity (AMSAA) or the U. S. Army Ballistics Research Laboratory (BRL). Depending on the weapon system, various degrees of protection may also be listed (hull defilade, fully exposed, overhead cover, etc.). For each firer-target engagement, the computer draws a random number and compares it to the  $P_k$  at that range and degree of protection. If the random number is less than or equal to the  $P_k$ , the target is assessed a casualty. Depending on the particular experiment, there may be several categories of "kills," such as firepower, mobility,

communications, or total. If the target is not assessed a casualty, he may or may not receive a message or signal that he was fired upon (He should still be able to detect the firing signature.). The degree of protection or attitude of the target weapon system can significantly affect the Pks. Often the degree of protection is determined based on another random number draw or by a subprogram based on speed and direction of movement of the target weapon system. It should be obvious that several "lucky" (or "unlucky") random number draws can drastically change the course of battle (and experimental results) and create a large variance in the number of casualties between repeated trials.

### C. SUMMATION OF PKS

The large variance in the number of casualties and the large number of repetitions of field exercises required to attain statistical stability prompted Dr. Bryson to search for a better method to measure casualty assessment. Dr. Bryson suggested that the summation of probabilities of kill (Pks) be used. The estimator based on the summation of Pks has a smaller variance than the estimator based on observed number of kills, when trying to estimate the expected number of casualties.

A simple example is when engagements  $X_1, X_2, X_3, \dots$  are independent, identically distributed (IID) Bernoulli random variables with an expected value of  $p$  (ie.  $X_1, X_2, X_3, \dots$  are IID  $b(1,p)$ ;  $E(X_i) = p$  for  $i = 1, 2, 3, \dots$ ). Here,  $X_i = 1$  if the  $i^{\text{th}}$  engagement results in a kill;  $X_i = 0$  otherwise. Suppose  $N$  is the (random) number of engagements in a trial. Then the number of casualties is  $X_1 + X_2 + \dots + X_N$ . The expected number of casualties is:

$$\begin{aligned} E\left(\sum_{i=1}^N X_i\right) &= E_N\left(E\sum_{i=1}^N X_i | N\right) \\ &= E_N(N * p) \\ &= \mu_N p . \end{aligned} \tag{2.1}$$

For a given battle, which will yield observations on  $N$  and  $X_1, X_2, \dots, X_N$ , two possible estimators for  $\mu_N p$  are:

$$\text{observed frequency: } \text{est}(\mu_N p) = \sum_{i=1}^N X_i . \tag{2.2}$$

and

$$\text{summation of Pks: } \text{est}'(\mu_N p) = \sum_{i=1}^N p = N * p. \quad (2.3)$$

Both of these estimators are unbiased, the first by Equation 2.1 and the second by the fact that, in Equation 2.3,  $E(N * p) = \mu_N p$ . A conditioning argument can be used to find the variance of the observed frequency estimator [Ref. 5: pp. 98-99]:

$$\begin{aligned} \text{Var}(\text{est}(\mu_N p)) &= E_N(\text{Var}(\sum_{i=1}^N X_i | N)) + \text{Var}_N(E(\sum_{i=1}^N X_i | N)) \\ &= E_N(N * p * q) + \text{Var}(Np) \\ &= pqE_N(N) + p^2 \text{Var}(N) \\ &= pq\mu_N + p^2(\sigma_N)^2. \end{aligned} \quad (2.4)$$

The variance of the summation of Pks estimator,  $\text{est}'(\mu_N p)$  is :

$$\begin{aligned} \text{Var}(\text{est}'(\mu_N p)) &= \text{Var}(Np) \\ &= p^2 \text{Var}(N) \\ &= p^2(\sigma_N)^2. \end{aligned} \quad (2.5)$$

For this example, the variance of the summation of Pks estimator is smaller than the variance of the observed frequency estimator ( $p^2(\sigma_N)^2 < pq\mu_N + p^2(\sigma_N)^2$ ). For example, if  $N$  were geometric( $p$ ), so  $\mu_N = 1/p$  and  $\sigma_N^2 = q/p^2$ , the observed frequency estimator could have twice the variance of the summation of Pks estimator ( $p^2\sigma_N^2 = q$  and  $pq\mu_N = q$ , so  $q < q + q$ ). [Ref. 6: pp.205-206]

The summation of Pks estimates the expected number of enemy casualties by summing the Pks of the friendly firers for each engagement. In mathematical notation,

$$E(K) = \sum_{i=1}^N Pk_i \quad (2.6)$$

where  $E(K)$  = Expected enemy kills,

$Pk_i$  = Probability of kill by the friendly firer, in the  $i^{th}$  engagement. and  
 $N$  = Total number of engagements (friendly firer-enemy target).

TABLE 1  
 SUMMATION OF PKS VERSUS SIMULATION

Eng No.	Firer and Pk	Simulated Kills		Pk Kills	
		Red	Blue	vs Red	vs Blue
1	Red .6		Survive		.6
2	Red .7		Kill		.7
3	Red .1		Survive		.1
4	Blue .8	Survive		.8	
5	Blue .3	Survive		.3	
TOTAL KILLS		0	1	1.1	1.4

A simple modified example from [Ref. 7: p. 7] has three Red aircraft attacking five Blue tanks with supporting air defense weapons. Suppose five engagements occur as shown in Table 1, and suppose the RTCA results in one Blue and zero Red casualties. The expected casualty count for each engagement is known since the Pks are known, so a partial kill (casualty) equal to the observed Pk should be credited for each engagement. Overall, expected casualties should be estimated by summing these expected casualties. In this example, the estimates of expected casualties would be 1.1 Red casualties and 1.4 Blue casualties.

It could be argued that there is an inherent problem with estimating casualties with the summation of Pks. That problem is that individual weapon systems or units may be estimated to be "killed" more than once. From our example, if the first two engagements were against the same target, Blue Tank A, then 1.3 casualties would be credited against Tank A. The initial reaction of some analysts is to deem such overkill as intolerable and attempt to modify the method of crediting casualties to ensure that no more than one kill is ever credited against a given target. There is at least one case in which limiting overkill is desirable. In most cases it is desirable to allow overkill to occur.

One situation in which overkill is clearly undesirable is when one weapon system fires several rounds in rapid succession. The firing of all the rounds should be considered only one engagement. The engagement  $P_k$  should be calculated using the appropriate product rather than the summation of  $P_k$ s. That is, if  $n$  rounds are fired each with a  $P_k = p$ , then  $1-(1-p)^n$  equation might be used rather than  $p$ , for a "burst  $P_k$ ". A modification of this formula would be necessary if independence of kill events, related to rounds within a burst, does not hold. As long as  $p$  is small, there is little difference between  $p$  and  $1-(1-p)^n$ ; but if  $p$  is large, the difference is not negligible. For example, if an anti-tank weapon fires  $n=3$  rounds at a tank at close range with  $P_k = 0.8$ , then  $(1-(1-p)^n) = 0.992$ . Some analysts have suggested using similar formulas, for each target, to avoid credited kills greater than one. If products of  $P_k$ s rather than the summation of  $P_k$ s were used to credit casualties, no more than one credited kill could ever be accumulated against a single weapon system. Crediting casualties using products is misleading because it generally underestimates the expected attrition as shown in [Ref. 7: pp. 9-12]. Crediting more than one kill against a single weapon system must be permitted if unbiased estimates of expected attrition are desired. As long as the RTCA is allowed to "shape the battlefield," some weapon systems will be removed from the experiment with less than a whole credited kill (casualty), so other players must be allowed to accumulate more than one credited kill of some targets to compensate for this shortfall.

#### **D. WHY IS ALIVENESS NEEDED?**

Real Time Casualty Assessment (RTCA) is used to "shape the battle" in Army operational tests by simulating attrition in near real time based on measured engagement conditions. As long as those attrition rates used in real time are "approximately correct," attrition rates suffice to "shape the battle." However, if test measures of effectiveness (MOEs) involve force losses, observed attrition rates that are only approximately correct may not be good enough. One goal of the operational test may be to estimate the expected losses had live ordnance been used. Aliveness analysis is a computational technique which was designed to attain this goal by crediting kills adjusted for the cumulative effects of differences between the actual probabilities of kill (PKA) and the used probabilities of kill (PKU). Aliveness analysis has several advantages over the use of the number of real time casualties or the use of summation of  $P_k$ s, in estimating expected casualties.

One disadvantage in using the number of real time casualties is that they may be inaccurate in some cases because instrumentation produces irregularities which cannot be resolved in real time. Post-test analysis can resolve some instrumentation irregularities. Post-test analysis of the battle may identify engagements that did not go to proper assessment because they were improperly recorded by test instrumentation or they were partially garbled during real time computer processing. This may be caused by instrumentation failure, faulty real-time position location data, or computer processing time-outs or buffer overflow. Such an analysis can produce, for each engagement, a list of the firer identification, the target identification, relevant firing conditions, and the Pk associated with that engagement. Once actual engagement conditions are determined post test, the actual probability of kill (PKA) can be computed and compared to the probability of kill used in the experiment (PKU). Missed engagements are modelled with  $PKU = 0.00$ . Whenever the two probabilities of kill differ, the attrition used during the experiment tends to be incorrect and may start a cascade of erroneous real time losses. This anomaly may be caused by faulty real-time position location data, software errors, or by errors in the Pk tables. Some problems may develop when one attempts to replay a simulation after the fact: What happens to an aircraft which was killed in real time, but survived in the post-test analysis? Or what should be done with tank A which was killed in real time by aircraft B, when during post-test it is determined that stinger C killed aircraft B before tank A was engaged? Another problem may be that the Pks may change post-test. It may be desired to modify the Pks, post-test, to conduct "what if" analyses, involving changes to the Pks. Because of these disadvantages, some users of attrition estimates believe that Real Time Casualty Assessment (RTCA) casualties are not suitable for analysis. [Ref. 5]

Although summation of Pks is a better estimator of mean attrition than using the casualties directly determined by the RTCA simulation, it is not perfect. A disadvantage of using the summation of Pks is that it cannot completely adjust to differences between PKUs and PKAs. For example, engagements with  $PKA > PKU$  would have left too many players on one side on the simulated battlefield and therefore resulted in too many engagements and casualties on the other side. Summation of Pks cannot adjust for the "excess" players, engagements, or casualties. A specific example is the post-test downward adjustment of the Pks of Blue anti-tank weapon systems against Red tanks, which should have resulted in fewer Red tank casualties and

therefore increased Blue attrition. However, such an increase in Blue force attrition could not be reflected in the summation of Pks because Red tank versus Blue tank Pks were unchanged.

Analysts can use aliveness analysis to provide sensible adjustments to casualty estimates by reducing or increasing credited casualties (kills) to compensate for cumulative errors in attrition [Ref. 7: p. 15]:

1. If  $PKU < PKA$ , then too little RTCA attrition was applied and the subsequent attrition capability of the target should be decreased, if it survives.
2. If  $PKU > PKA$ , then too much RTCA attrition was applied and the subsequent attrition capability of the target should be increased, if it survives.
3. If  $PKU = PKA$ , then the RTCA attrition was correct and no adjustment should be applied.
4. Missed engagements fall into category 1,  $PKU < PKA$  where  $PKU = 0.00$  and the subsequent attrition capability of the target should be decreased.
5. If the target is killed during RTCA simulation, it is removed from play and no adjustment need be applied.

#### E. ALIVENESS FORMULAS

Looking for a better estimator than summation of Pks for the expected number of casualties in an RTCA experiment, Dr. Marion R. Bryson of CDEC and Dr. Carl T. Russell of OTEA developed "aliveness analysis". Aliveness analysis adjusts for the differences between real time and post-test probabilities of kill (and resulting attrition rates) by crediting partial kills via "potency" or "aliveness" weights on live players. Each player possesses an aliveness factor A where  $A_{\text{initial}} = 1.0$  for all players. Cumulative credited kills by player I (firer) versus player J (target),  $K(I,J)$ , are tracked, at each engagement of J by I, with  $K_{\text{initial}}(I,J) = 0.0$  for all player pairs. Dr. Bryson and Dr. Russell started with the formula [Ref. 1]:

$$K(I,J) = Pk * A(I) * A(J) * F \quad (2.7)$$

where F is an unknown factor.

It was not clear how one should adjust the aliveness of the target (  $A(J)$  ) after each engagement. Using empirical observations, an early aliveness formula was suggested:

$$A_{\text{new}}(J) = A_{\text{old}}(J) * (1 - A(I) * PKA) (1-PKU). \quad (2.8)$$

This is a simple equation to use, but it allows players to attain negative aliveness under some conditions. If the computed new aliveness of target J is negative, only the current value of  $A_{old}(J)$  kills would be credited and  $A_{new}(J)$  would be reduced to zero.

Another aliveness formula, which corrected that problem, was suggested:

$$A_{new}(J) = A_{old}(J) * (1-PKA) (1-PKU). \quad (2.9)$$

This is not an aesthetically pleasing equation because it completely ignores the aliveness of the firer ( $A(I)$ ). That means that a target's aliveness ( $A(J)$ ) will be decremented an identical amount if J is engaged by a firer whose aliveness is one (1.0) or one tenth (0.1). [Ref. 1]

After more empirical work, Dr. Bryson and Dr. Russell suggested a final set of aliveness equations. Suppose a player I (firer with potency  $A_{old}(I)$ ) engages player J (target with potency  $A_{old}(J)$ ), with probability of kill PKA (actual Pk for post-test analysis) and where the probability of kill used in the RTCA is PKU. The aliveness factors and the cumulative credited kills are computed as follows:

$$K_{new}(I,J) = K_{old}(I,J) + A_{old}(J) * (1 - (1-PKA) A_{old}(I)), \quad (2.10)$$

$$A_{new}(I) = A_{old}(I). \quad (2.11)$$

$$A_{new}(J) = A_{old}(J) * (1-PKA) A_{old}(I) (1-PKU). \quad (2.12)$$

The underlying motivation for these formulas is straight-forward. If the firer has an aliveness of 1.0 ( $A_{old}(I) = 1.0$ ), the calculation adjusts the potency of the surviving players as a ratio of survival probabilities. That is,

- If a player survives with twice the probability that he should have, his potency is halved. For example, if  $PKA = 0.6$  and  $PKU = 0.2$ , then

$$(1-PKA) (1-PKU) = (1.0 - 0.6) (1.0 - 0.2) = 0.4 * 0.8 = 0.5. \quad (2.13)$$

- If a player survives with half the probability that he should have, his potency is doubled. For example, if  $PKA = 0.6$  and  $PKU = 0.8$ , then

$$(1-PKA)/(1-PKU) = (1.0 - 0.6)/(1.0 - 0.8) = 0.4/0.2 = 2.0. \quad (2.14)$$

The exponential adjustment for the credited kills in equation 2.10 is based on a standard statistical formula:

- $n$  independent firings with  $P_k = p$  gives a total  $P_k = 1 - (1-p)^n$ .
- If the potency,  $A(I)$ , of the firer is treated as a "shot multiplier", analogous to  $A(I) = n$ , a player with potency  $A(I)$  firing with  $P_k = p$  gives a total probability of kill,  $P_k = 1 - (1-p)^n = 1 - (1-p)^{A(I)}$ .
- If  $I$  kills a target  $J$  with aliveness  $A(J)$ ,  $I$  is credited with  $A(J)$  kills.

The calculation of equation 2.10 reduces to the summation of  $P_k$ s when the  $PKA$ s =  $PKU$ s. Suppose that a simulation begins with  $PKA = PKU = 0.6$ . And suppose the aliveness of both the firer and target are identical,  $A(I) = A(J) = 1.00$ . The resultant credited kill is computed as follows:

$$\begin{aligned} K_{\text{new}}(I,J) &= K_{\text{old}}(I,J) + A_{\text{old}}(J) * (1 - (1-PKA)^{A_{\text{old}}(I)}) \\ &= 0.00 + 1.00 * (1 - (1 - 0.60)^{1.00}) \\ &= 0.60. \end{aligned} \quad (2.15)$$

This result, 0.60, is the same amount that would have been credited as a kill using the summation of  $P_k$ s. The calculation always adjusts in the right direction when  $A(I) = 1.0$  and performs well in practice. [Ref. 9]

## F. EXAMPLES OF ALIVENESS COMPUTATIONS

A good way to examine aliveness calculations is to follow how the aliveness analysis performs on an actual or hypothetical sequence of engagements. The following is a hypothetical example of a tank versus anti-tank experiment consisting of two Blue anti-tank weapon systems (such as TOWs) engaging one Red tank. All three weapon systems (AT #1, AT #2, and Red tank #1) have aliveness values ( $A_{\text{old}}$ ) of 1.0 at the beginning of the trial. The  $PKU$  used in the first engagement between AT #1 and Red tank #1 was 0.83 when the  $PKA$  calculated post-trial was 0.61. The initial conditions and preliminary calculations are:

- I = AT #1  $A_{old}(I) = 1.00$
- J = Red tank #1  $A_{old}(J) = 1.00$
- PKA = 0.61  $1 - PKA = 0.39$
- PKU = 0.83  $1 - PKU = 0.17$
- $(1-PKA)^{A_{old}(I)} = 0.39$   $K_{old}(I,J) = 0.00$

Suppose that in the RTCA simulation, Red tank #1 survived this engagement. Substituting the above values into equations 2.10, 2.11, and 2.12 result in the following:

$$\begin{aligned}
 K_{new}(I,J) &= K_{old}(I,J) + A_{old}(J) * (1 - (1-PKA)^{A_{old}(I)}) \\
 &= 0.00 + 1.00 * (1 - (0.39)^{(1.00)}) = 0.61 \\
 A_{new}(I) &= A_{old}(I) = 1.00 \\
 A_{new}(J) &= A_{old}(J) * (1-PKA)^{A_{old}(I)} / (1-PKU) \\
 &= 1.00 * (0.39)^{(1.00)} / 0.17 = 2.29
 \end{aligned}
 \tag{2.16}$$

The potency of target J, Red tank #1, is increased 2.29 times. One interpretation of the increased potency is that in a large number of engagements using the PKA, there should be 2.29 times as many survivors as were observed in real time using the PKU. The aliveness calculation credits 0.61 kills against Red tank #1 and increases the potency of Red tank #1 to 2.29.

In the second engagement, AT #2 engages Red tank #1. This engagement was not assessed in real time. Reasons for the non-assessment could be many: computer malfunction, "what-if" analysis if AT #2 was in a different position and able to engage Red tank #1, etc.. Since no engagement was performed, PKU = 0.00. The PKA established post-test was 0.34. The aliveness of AT #2 is still 1.00, but the aliveness of Red tank #1 from the first engagement is now 2.29. The initial conditions and preliminary calculations for the second engagement of the experiment are:

- I = AT #2  $A_{old}(I) = 1.00$
- J = Red tank #1  $A_{old}(J) = 2.29$
- PKA = 0.34  $1 - PKA = 0.66$
- PKU = 0.00  $1 - PKU = 1.00$
- $(1-PKA)^{A_{old}(I)} = 0.66$   $K_{old}(I,J) = 0.00$

Substituting the above values into equations 2.10, 2.11, and 2.12 result in the following:

TABLE 2  
ANTI-TANK VERSUS TANK EXAMPLE

Eng No.	Firer and Potency	Engagements		New Pot.	Cred Kills	Sim Kills
		Target and Potency	PKA PKU			
1.	AT #1, 1.00	Tank #1, 1.00	0.61 0.83	2.29	0.61	Survive
2.	AT #2, 1.00	Tank #1, 2.29	0.34 0.00	1.51	0.78	Survive*

\* not assessed

Summary of Casualty Estimation

	Sim Kills	Sum of Pks	Cred Kills
Against Red	0.00	0.95	1.39
Against Blue	0.00	0.00	0.00

$$K_{\text{new}}(I,J) = K_{\text{old}}(I,J) + A_{\text{old}}(J) * (1 - (1-PKA)^{A_{\text{old}}(I)})$$

$$= 0.00 + 2.29 * (1 - (0.66)^{(1.00)}) = 0.78$$

$$A_{\text{new}}(I) = A_{\text{old}}(I) = 1.00 \quad (2.17)$$

$$A_{\text{new}}(J) = A_{\text{old}}(J) * (1-PKA)^{A_{\text{old}}(I)} / (1-PKU)$$

$$= 2.29 * (0.66)^{(1.00)} / 1.00 = 1.51 .$$

In this engagement, the actual survival probability (1-PKA) is 0.66 times what was applied in real time. That means in a large number of engagements with the probability of kill equal to 0.34 (Pk = 0.34), there should be 0.66 times as many survivors as were observed in real time with the Pk = 0.00. The aliveness calculation decreased the potency of Tank #1 to 1.51 = (0.66 \* 2.29) and credits AT #2 with 0.78 = 2.29 \* 0.34 kills against Red tank #1. In this short two engagement example, 1.39 kills are credited by aliveness, while the summation of Pks estimation of casualties is 0.95 and no RTCA casualties were observed (see Table 2).

#### G. EXAMPLES FROM THE SGT YORK FOLLOW ON EVALUATION (FOE)

The force-on-force portion of the SGT York Follow on Evaluation (FOE) was conducted at Fort Hunter Liggett, California, from 2 April until 22 May 1985. The FOE was a platoon-level evaluation conducted to compare the capabilities of three

different configurations of air defense weapon systems to provide air protection to an armor task force (battalion-size element) in similar types of missions. The main mission performance criteria for the evaluations was the proportion of Blue force losses to Red air during trials when the three different air defense configurations were present. During the evaluation there were frequent differences between the PKAs and the PKUs. The most common case ( $PKA > PKU = 0.00$ ) was when engagements did not go to real time assessment, but engagement conditions (and PKAs) were recovered through post-test analysis (examination of video and audio recordings, etc.). During the evaluation, it was estimated that forty to fifty percent of the engagements did not go to real time assessment. An adjustment such as aliveness analysis may be desirable in such a case.

Dr. Carl T. Russell has presented several briefings on the aliveness calculations based on the SGT York testing (see [Refs. 7,9]). He used fictitious probabilities of kill values to keep his briefings unclassified. However, the aliveness calculations themselves closely resemble the calculations obtained from the actual engagement sequences (see [Ref. 7: pp. 17-19]). This first example consists of a series of four engagements involving SGT York #1. The first engagement against Fitter #1 is similar to the computations in the previous example.

- $I = \text{SGT York \#1}$                        $A_{\text{old}}(I) = 1.00$
- $J = \text{Fitter \#1}$                                $A_{\text{old}}(J) = 1.00$
- $PKA = 0.26$                                    $1 - PKA = 0.74$
- $PKU = 0.54$                                    $1 - PKU = 0.46$
- $(1-PKA)A_{\text{old}}(I) = 0.74$                $K_{\text{old}}(I,J) = 0.00$

Substituting the above values into equations 2.10, 2.11, and 2.12 result in the following:

$$\begin{aligned}
 K_{\text{new}}(I,J) &= K_{\text{old}}(I,J) + A_{\text{old}}(J) * (1 - (1-PKA)A_{\text{old}}(I)) \\
 &= 0.00 + 1.00 * (1 - (0.74)^{(1.00)}) = 0.26 \\
 A_{\text{new}}(I) &= A_{\text{old}}(I) = 1.00 \\
 A_{\text{new}}(J) &= A_{\text{old}}(J) * (1-PKA)A_{\text{old}}(I) (1-PKU) \\
 &= 1.00 * (0.74)^{(1.00)} 0.46 = 1.61 .
 \end{aligned}
 \tag{2.18}$$

The potency of target J, Fitter #1, increases 1.61 times. Again, one interpretation of this is that in a large number of engagements, there should be 1.61 times as many

survivors as were observed in real time. One big difference in this engagement is that the target, Fitter #1, was killed in RTCA and its increased aliveness has no impact on subsequent engagements.

SGT York #1 was the target of Hind #3 in the second engagement. SGT York #1 should have survived with a probability of 0.28 (1-PKA), but since the engagement did not go to assessment an expected surviving value of 1.00 was applied.

- I = Hind #3  $A_{old}(I) = 1.00$
- J = SGT York #1  $A_{old}(J) = 1.00$
- PKA = 0.72  $1 - PKA = 0.28$
- PKU = 0.00  $1 - PKU = 1.00$
- $(1-PKA)^{A_{old}(I)} = 0.28$   $K_{old}(I,J) = 0.00$

Substituting the above values into equations 2.10, 2.11, and 2.12 result in the following:

$$\begin{aligned}
 K_{new}(I,J) &= K_{old}(I,J) + A_{old}(J) * (1 - (1-PKA)^{A_{old}(I)}) \\
 &= 0.00 + 1.00 * (1 - (0.28)^{(1.00)}) = 0.72 \\
 A_{new}(I) &= A_{old}(I) = 1.00 \\
 A_{new}(J) &= A_{old}(J) * (1-PKA)^{A_{old}(I)} / (1-PKU) \\
 &= 1.00 * (0.28)^{(1.00)} / 1.00 = 0.28 .
 \end{aligned}
 \tag{2.19}$$

The aliveness calculation decreases the potency of SGT York #1 to 0.28 and credits 0.72 kills against it.

Now SGT York #1, with a decreased potency of 0.28, engages Fitter #3 in the third engagement. Fitter #3 has an aliveness of 1.94. The expected surviving value of the target should be greater than 1-PKA = 0.69 because the firer is only "partially alive." That means if this trial was conducted many times in a perfect RTCA environment, SGT York #1 would only be around this long to engage targets a fraction of the time.

- I = SGT York #1  $A_{old}(I) = 0.28$
- J = Fitter #3  $A_{old}(J) = 1.94$
- PKA = 0.31  $1 - PKA = 0.69$
- PKU = 0.49  $1 - PKU = 0.51$
- $(1-PKA)^{A_{old}(I)} = 0.90$   $K_{old}(I,J) = 0.00$

Substituting the above values into equations 2.10, 2.11, and 2.12 result in the following:

$$\begin{aligned}
 K_{\text{new}}(I,J) &= K_{\text{old}}(I,J) + A_{\text{old}}(J) * (1 - (1-PKA)^{A_{\text{old}}(I)}) \\
 &= 0.00 + 1.94 * (1 - (0.69)^{(0.28)}) = 0.19 \\
 A_{\text{new}}(I) &= A_{\text{old}}(I) = 0.28 \\
 A_{\text{new}}(J) &= A_{\text{old}}(J) * (1-PKA)^{A_{\text{old}}(I)} / (1-PKU) \\
 &= 1.94 * (0.69)^{(0.28)} / 0.51 = 3.42 .
 \end{aligned}
 \tag{2.20}$$

The aliveness formula computes the expected surviving value of the target to be  $(1-PKA)^{A_{\text{old}}(I)} = 0.90$  and therefore increases the potency of the target 1.77 times to 3.42. The credited kill was only 0.19 as shown above. Once a firer has an aliveness value less than 1.00, not only are credited kills reduced, but the potency of targets tend to increase.

In the last engagement of this example, SGT York #1 engaged Hind #3. There was no change in the probability of kill ( $PKU = PKA$ ), but this engagement demonstrates what can happen when the aliveness of the firer is less than 1.00.

- I = SGT York #1	$A_{\text{old}}(I) = 0.28$
- J = Hind #3	$A_{\text{old}}(J) = 1.00$
- PKA = 0.25	$1 - PKA = 0.75$
- PKU = 0.25	$1 - PKU = 0.75$
- $(1-PKA)^{A_{\text{old}}(I)} = 0.92$	$K_{\text{old}}(I,J) = 0.00$

Substituting the above values into equations 2.10, 2.11, and 2.12 result in the following:

$$\begin{aligned}
 K_{\text{new}}(I,J) &= K_{\text{old}}(I,J) + A_{\text{old}}(J) * (1 - (1-PKA)^{A_{\text{old}}(I)}) \\
 &= 0.00 + 1.00 * (1 - (0.75)^{(0.28)}) = 0.08 \\
 A_{\text{new}}(I) &= A_{\text{old}}(I) = 0.28 \\
 A_{\text{new}}(J) &= A_{\text{old}}(J) * (1-PKA)^{A_{\text{old}}(I)} / (1-PKU) \\
 &= 1.00 * (0.75)^{(0.28)} / 0.75 = 1.23 .
 \end{aligned}
 \tag{2.21}$$

The potency of the target increased 23 percent (1.00 to 1.23) even though the probabilities of kill were unchanged. Since the target was assessed a casualty and removed from the experiment during the RTCA, the increased potency had no effect

on subsequent engagements. Only 0.08 of a kill was credited against the target because of the small aliveness value of the firer. The summary of this example is given in Table 3. The aliveness calculations credited 0.53 kills against Red forces while the summation of Pks resulted in 0.82 casualties and there were 2.0 simulation kills. The aliveness results make good intuitive sense again. The two simulated kills were assessed because of "lucky" random number draws. The summation of Pks does not consider the increased potency of the targets or the degraded potency of the firer, SGT York #1. The aliveness technique includes the potency of both the firer and target in the computations and therefore produces more acceptable estimates of expected casualties.

TABLE 3  
SGT YORK FOE EXAMPLE - SGT YORK #1

Eng No.	Engagements		PKA	PKU	New Pot.	Cred Kills	Sim Kills
	Firer and Potency	Target and Potency					
1.	York #1, 1.00	Fit #1, 1.00	0.61	0.83	1.61	0.26	Kill
2.	Hind #3, 1.00	York #1, 1.00	0.34	0.00	0.28	0.72	Survive*
3.	York #1, 0.28	Fit #3, 1.94	0.61	0.83	3.42	0.19	Survive
4.	York #1, 0.28	Hind #3, 1.00	0.25	0.25	1.23	0.08	Kill

\* not assessed

Summary of Casualty Estimation

	Sim Kills	Sum of Pks	Cred Kills
Against Red	2.00	0.82	0.53
Against Blue	0.00	0.72	0.72

The last example is summarized in Table 4. It is a listing of the engagements that involved Hind #2 as the firer. This is a fairly routine example since the firer's aliveness is 1.00 and the PKUs were either correct or were 0.00 if the engagement was not resolved during RTCA. In all but one instance, the credited kill was equal to the PKA and all three measures of attrition were nearly equal. As the summary of attrition estimates shows, the number of aliveness credited casualties fell between the simulated kills and number of casualties estimated by the summation of Pks. In fact, this same ordering occurred in all but three of the SGT York fifty-two trials [Ref. 7: p.

TABLE 4  
SGT YORK FOE EXAMPLE - HIND #2

Target	PKA	PKU	Aliveness	Crt'd Kill	Result
Abrams #10	0.57	0.57	1.00 → 1.00	0.57	Kill
Abrams #5	0.38	0.38	1.00 → 1.00	0.38	Survive
Abrams #13	0.45	0.45	1.00 → 1.00	0.45	Kill
Abrams #13	0.00	0.00	1.00 → 1.00	0.00	Dead Tgt
Abrams #7	0.46	0.00	1.00 → 0.54	0.46	N/A
Abrams #5	0.00	0.00	1.00 → 1.00	0.00	Dead Tgt
Unknown	0.00	0.00	1.00 → 1.00	0.00	N/A
Abrams #14	0.51	0.51	1.00 → 1.00	0.51	Survive
SGT York #4	0.95	0.00	1.00 → 0.05	0.95	N/A
Abrams #16	0.39	0.00	1.00 → 0.61	0.39	N/A
Abrams #16	0.48	0.48	0.57 → 0.57	0.28	Kill
Bradley #10	0.72	0.72	1.00 → 1.00	0.72	Kill
Abrams #16	0.00	0.00	0.57 → 0.57	0.00	Dead Tgt

Summary of Attrition Estimates

Simulated Kills Against Blue	= 4.00
Summation of Pks Against Blue	= 4.91
Credited Kills Against Blue	= 4.71

19, Figure 6]. This occurred because the most common RTCA error for the trials was an engagement failing to go to real time assessment. When the  $PKA > PKU = 0.00$ , no simulated kills were produced and the survivor's aliveness became less than 1.00, which will make the total aliveness credited casualties less than the number of estimated casualties by the summation of Pks. So the number of estimated casualties by the summation of Pks becomes an upper bound on the aliveness calculations of credited kills. Since the summation of Pks simply sums the Pks regardless of the potency of the firer, it will always credit too many kills to a partially alive firer (a firer whose potency is less than 1.00). Another interpretation of the degraded potency is that in the long run, fewer survivors of that weapon system would be on the battlefield to initiate the engagement. Again, the preferred technique to estimate attrition appears to be the aliveness technique.

### III. CONDUCT OF THE SIMULATION

#### A. THE ALIVENESS PROGRAM

The aliveness techniques seem to work, but how sensitive are the aliveness techniques to changes in the parameters, such as target selection method or probability of kill? A series of simulations with varying parameters might reveal tendencies of the aliveness techniques to be biased under certain circumstances. A simple battle simulation program, useful for evaluating aliveness adjustments, was obtained from Dr. Russell. A modified version of this program is listed in Appendix A.

The program includes several prompts for input information. For both the blue and red sides, the user inputs the number of players; the probability of kill used in the simulation, PKU; the probability of kill used in the aliveness adjustments, i.e. actual probability of kill, PKA; and the amount of jitter desired in the probabilities of kill. The jitter input varies the probabilities of kill about the inputted PKAs and PKUs. The program generates output on the results of every engagement using all three aliveness methods (the current method and two earlier versions). The program conducts a battle, hereafter called an iteration, until all of the players on one side or the other are eliminated. Optional summaries are available after each full page of engagement output in addition to summaries at the end of each iteration.

The program makes extensive use of the random number generator available in the Microsoft BASIC computer language, which may be a shortcoming. The pseudo-random number generator provided with this BASIC, used on many microcomputers which use Microsoft DOS, has serious shortcomings [Ref. 10]. Random number draws determine many events in simulated battles, the program such as which side will fire next. If the random number draw is less than 0.50, a blue firer will engage a red target (player); if not a red firer will engage a blue target (player). Further random number draws determine which particular firer (player) will engage which particular target (player) of the surviving players. A final random number draw determines the outcome of the engagement. Random number draws are also used to jitter the probabilities of kill, if that option is selected.

The program used to analyze the sensitivity of aliveness techniques to changes in the parameters is a modification of the original program. The termination condition

for each iteration is the complete elimination of one side. The original program required three iterations for each set of initial data. Some of the changes in the modified program included conducting ten iterations per set of initial conditions and the elimination of some output such as page summaries and the results from each particular engagement.

## **B. EXPERIMENTAL SET-UP**

The sensitivity of aliveness adjustments to changes in parameters was examined in a realistic scenario involving a company sized attacker engaging a platoon in the defense. Since the attacker tries to maintain a three to one advantage, a scenario was developed in which a red company consisting of twelve players attacks a blue platoon consisting of four players. Although the probabilities of kill are dependent on many factors such as range, cover, target exposure, target orientation, target direction and speed of movement, etc.; it is usually conceded that a dug-in defender has the advantage when engaging like forces. A "typical" probability of kill of the blue players against the red players of 0.50 was selected and the "typical" probability of kill of the red players against the blue players was selected to be 0.20.

The first parameter to be changed, in our examination of the sensitivity of the aliveness method, were the probabilities of kill. The PKAs and the PKUs of both the red and the blue players were changed in a systematic manner. The standard pair was comprised of a blue probability of kill (BPK) of 0.50 and a red probability of kill (RPK) of 0.20. Only one of the four probabilities of kill (BPKA, BPKU, RPKA, or RPKU) was changed at a time, so all simulations were constructed against the standard pair of probabilities of kill. For example, if the probabilities of kill used in a simulation were 0.75 (BPKU) and 0.20 (RPKU), the actual probabilities of kill (PKAs) would be the standard pair. A "complementary" simulation run would then be conducted with actual probabilities of kill of 0.75 (BPKA) and 0.20 (RPKA) against the standard pair of used probabilities of kill (PKUs). The complementary runs allowed comparison of aliveness results against simulation results for the same respective probabilities of kill.

## **C. CONDUCT OF THE FIRST EXPERIMENT**

The first experiment examined the sensitivity of aliveness adjustments to changes in the probabilities of kill and consisted of twenty-eight simulations using a modified aliveness program (listed in Appendix A). Fourteen probability of kill pairs were used

TABLE 5  
PROBABILITY OF KILL PAIRINGS  
FIRST EXPERIMENT

Changes in BPK		Changes in RPK	
BPK	RPK	BPK	RPK
0.25	0.20	0.50	0.00
0.40	0.20	0.50	0.10
0.45	0.20	0.50	0.15
0.55	0.20	0.50	0.25
0.60	0.20	0.50	0.30
0.75	0.20	0.50	0.40
0.50*	0.20*	0.50	0.50

\*standard Pk pair

as listed in Table 5. Each Pk pair was alternately substituted for the actual and used probabilities of kill and "run" against the standard Pk pair in a simulation. The probabilities of kill were jittered 0.05. For each engagement, the random method of firer and target selection was used. A random number draw determined which side would fire (e.g., Blue). A second random number draw determined which of the surviving players from the firing side (e.g., Blue) would fire, and a third random number draw determined which of the surviving players on the other side (e.g., Red) would be the target. Ten iterations were conducted for each simulation, for a total of two hundred and eighty iterations. In this experiment the simulated casualties were considered "ground truth" and the aliveness techniques were used to adjust the different probability of kill pairings to the standard pair.

STATGRAPHICS, a statistical graphics system designed for micro-computers, was used to summarize and analyze the data from the simulations [Ref. 11]. Some of the summary statistics for the first experiment are listed in Table 6. The results from the aliveness adjustments compared favorably with the actual simulation results. The estimated expected numbers of casualties were very close for the two methods, differing by less than five percent. The expected reduction in variance (and hence, standard

TABLE 6  
SUMMARY STATISTICS FOR THE FIRST EXPERIMENT

Statistic	Blue Cas Simulation	Blue Cas Aliveness	Red Cas Simulation	Red Cas Aliveness
Sample size	140	140	140	140
Average	3.43	3.39	8.21	8.57
Median	4	3.32	9	8.85
Variance	0.92	1.11	13.82	8.53
Standard Deviation	0.96	1.05	3.72	2.97
Minimum Value	0	0.95	0	0.00
Maximum Value	4	7.1	12	16.16
Range	4	6.15	12	16.16
Lower Quartile	3	2.875	5	6.60
Upper Quartile	4	4.05	12	10.57

deviation) was seen in the aliveness adjusted estimates for the number of red casualties, but not in the aliveness adjusted estimates for the number of blue casualties. Although the standard deviation of the aliveness adjusted estimates was higher for estimating the expected blue casualties, the increase was less than ten percent. One of the reasons that the standard deviation of the number of blue casualties by simulation was smaller could be that the number of blue simulated casualties is bounded. The number of simulated blue casualties can never exceed four, the number of blue players. As the number of players per side increases, so does the possible range of simulated casualties and the standard deviation should generally increase. The standard deviation of the estimated number of red casualties was lower for the aliveness techniques by over twenty percent. The frequency histograms for the first experiment are displayed in Appendix B. The histograms reflect that the simulation data is in discrete units and the aliveness data is continuous. The blue simulation data is stair-stepped in an increasing manner. All four blue players were killed in a majority (ninety-three of one hundred forty, or sixty-six percent) of the iterations. The red simulation data appears uniformly distributed except for the forty-seven iterations during which all of the red players were killed. The aliveness data for both the red and blue sides appears "normally" distributed. There appears to be some correlation between the magnitude

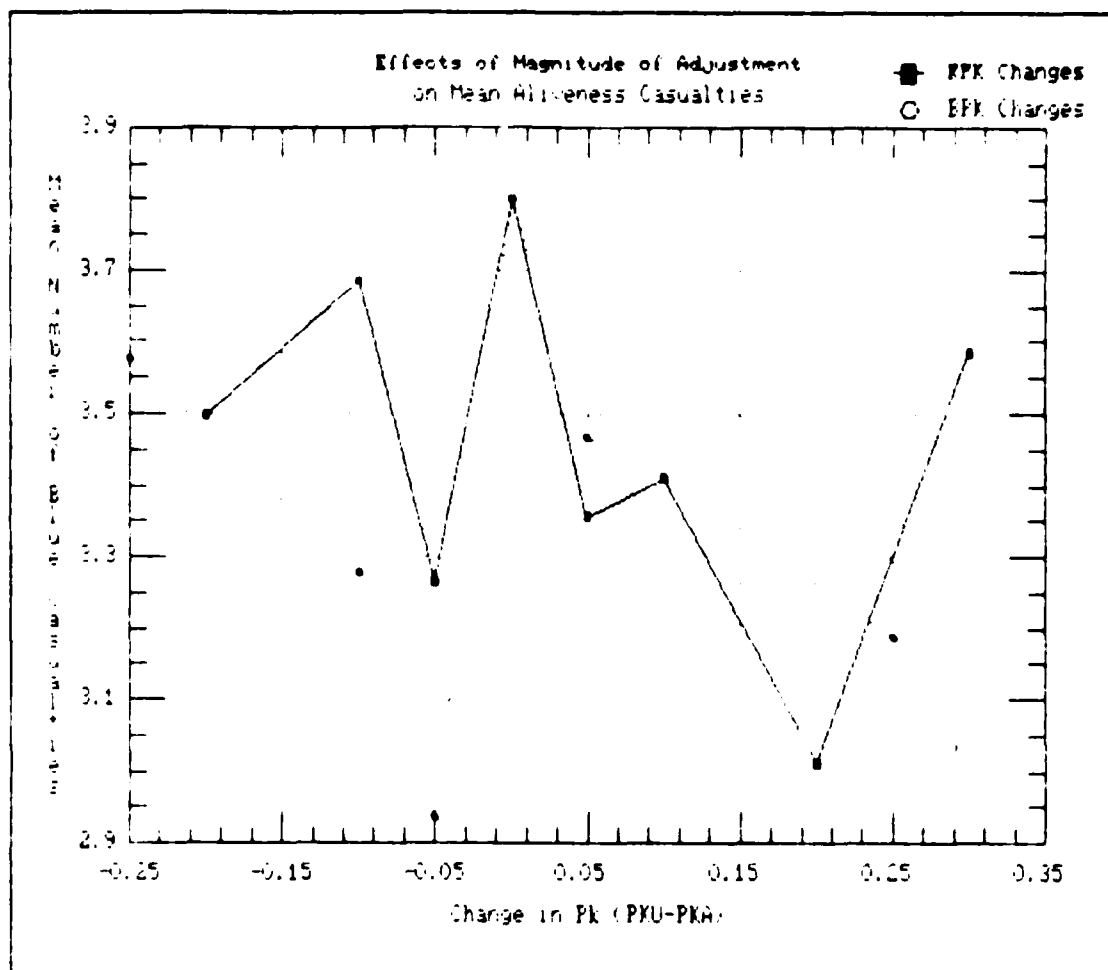


Figure 3.1 Blue Casualties by Aliveness  
as a Function of Change in Probability of Kill (PKU-PKA).

of the probability of kill adjustments and the estimated number of casualties using the aliveness method. The mean number of blue casualties from ten iterations plotted against the magnitude of change in probability of kill is shown in Figure 3.1. Only one probability of kill (red or blue) in each pair was changed at a time and that adjustment was always to the standard actual probability of kill (PKA) pair. The abscissa (x-axis) is the change in the probability of kill. The actual probability of kill (PKA) is subtracted from the used probability of kill (PKU). For example, if  $BPKU = 0.45$  and  $BPKA = 0.50$ , then the change in Pk is  $-0.05$ . If the standard probability of kill pair was used for both the simulation and the aliveness method, the change in Pk would be  $0.00$ . Both lines go through the same point when the change in Pk is  $0.00$  ( $PKU =$

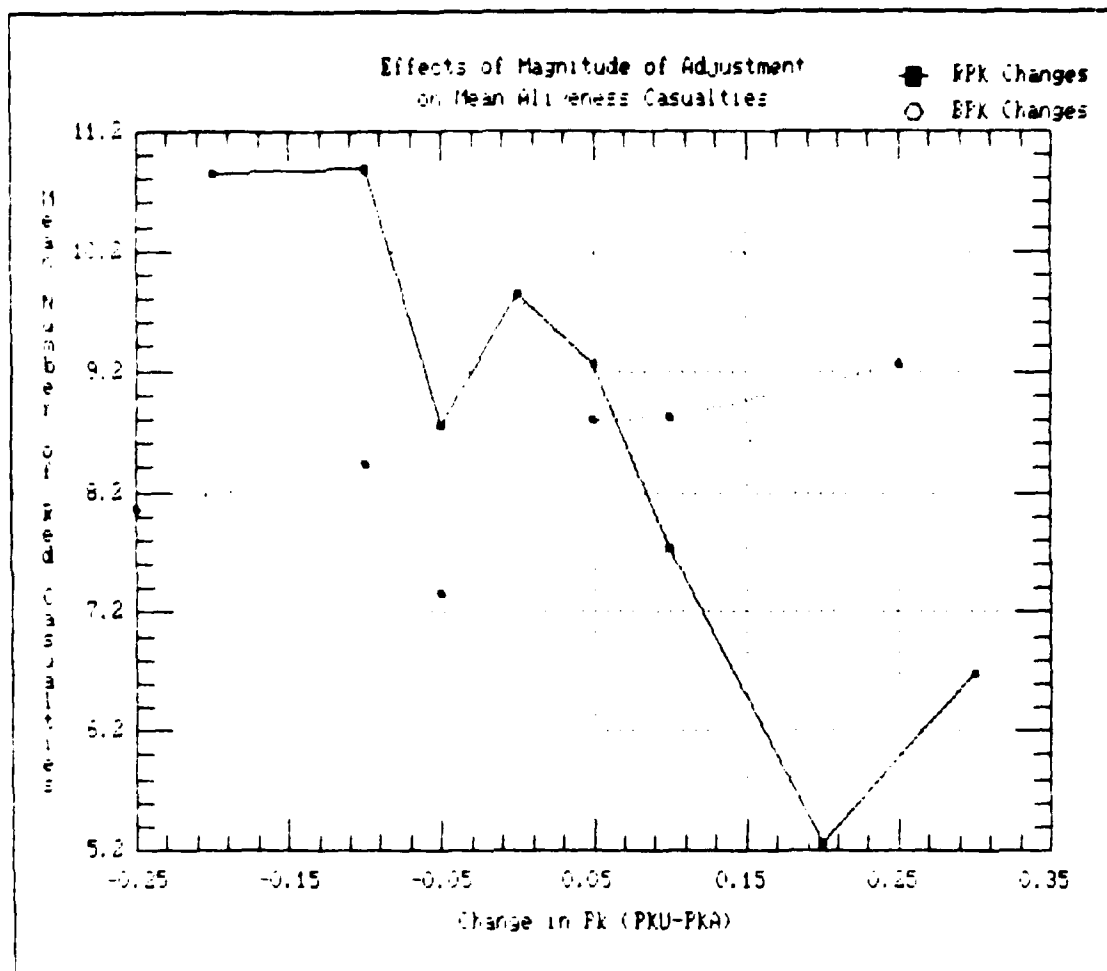


Figure 3.2 Red Casualties by Aliveness  
as a Function of Change in Probability of Kill (PKU-PKA).

PKA) because there was no change in either the blue or red probabilities of kill. The legend is in the top right corner of the figure. It is difficult to discern a clear pattern from the graph because of the variability of the means, however, it does appear that the solid line (changes in the red Pk) is higher for low values of the change of probability of kill than the dotted line (changes in blue Pk) and lower for the high values in the change of probability of kill. The small number of blue players and the susceptibility of engagement outcomes to the random number draws may explain some of the variability. There appears to be a more discernable pattern in the mean number of red casualties from ten iterations plotted against the magnitude of change in the probabilities of kill as shown in Figure 3.2. The solid line (changes in the red Pk) is

higher for low values of the change of probability of kill than the dotted line (changes in blue  $P_k$ ) and lower for the high values in the change of probability of kill. The minimum value for the number of red casualties using the aliveness method (0.00) was unexpected since a partial kill is credited for every engagement. The aliveness adjustment method was a "victim" of the random number draw in one specific iteration because there were only seven engagements in the iteration and the red side was selected as the firer for every engagement. Therefore for that iteration, the blue side suffered all of the credited casualties and the red side suffered none.

#### **D. CONDUCT OF THE SECOND EXPERIMENT**

A second experiment was conducted to examine the sensitivity of aliveness adjustments to changes in target selection methods. It consisted of thirty-six simulations using four modified aliveness programs. Four methods of selecting targets were used. The same random number draw procedure of selecting a side and a particular player to fire, that was used in the first experiment, was used in the second experiment. The first of the four methods of target selection was the random selection method used in the first experiment. The second target selection method chose as the target the surviving player with the highest aliveness factor. This method supposed that the firer would select targets in a way correlated with factors affecting their aliveness. The firer selected the surviving player with the highest aliveness value, perhaps a "superplayer" whose aliveness value was disproportionate to the other players. The target selection portion of the aliveness program for this method is listed in Appendix C. The third target selection method chose as the target the surviving player that the firer had the greatest probability of killing (the firer's highest  $PKU$ ). A large random jitter factor (0.25) was applied to the probabilities of kill to strongly test the accuracy of the aliveness adjustments. It has been suggested that soldiers in combat may often use this target selection method. The target selection portion of the aliveness program for this method is listed in Appendix D. The fourth target selection method chose as the target the surviving player which was most dangerous to the firer (the target with the highest  $PKU$ ). Again, a large random jitter factor (0.25) was applied to the probabilities of kill. This target selection method might require the most training and discipline in combat. The target selection portion of the aliveness program for this method is listed in Appendix E.

There were some indications during the first experiment that the distance or amount of adjustment to which the aliveness technique is applied has some impact on

TABLE 7  
PROBABILITY OF KILL PAIRINGS  
SECOND EXPERIMENT

PK PAIR	Simulation		Aliveness	
	BPKU	RPKU	BPKA	RPKA
1	0.50*	0.20*	0.50*	0.20*
2	0.50*	0.20*	0.25	0.20
3	0.25	0.20	0.50*	0.20*
4	0.50*	0.20*	0.50	0.10
5	0.50	0.10	0.50*	0.20*
6	0.50*	0.20*	0.50	0.30
7	0.50	0.30	0.50*	0.20*
8	0.50*	0.20*	0.75	0.20
9	0.75	0.20	0.50*	0.20*

\*standard Pk pair

the accuracy of the aliveness adjustments. For that reason, changes in the probability of kill pairs were also included in the second experiment. Five probability of kill pairs were used to conduct nine simulations for each of the target selection methods. A simulation using the standard pair for both the simulation and aliveness adjustments was conducted as a control. Each of the other four probability of kill pairs were alternately substituted for the probabilities of kill used in the simulation (PKUs) and the aliveness adjustments (PKAs) and run against the standard pair. Each simulation was given a PK pair number. The PK pair number and the corresponding probability of kill pairs are listed in Table 7.

STATGRAPHICS was used to summarize and analyze the data from the second experiment. The multifactor analysis of variance (ANOVA) procedure was used to analyze the effect of two qualitative factors, target selection method and probability of kill pair (TGTSELMETH and PKPAIR), on a response variable with one covariate, number of engagements (ENGAGE). Seven response variables were examined. Interactions between the two qualitative factors were also examined during each

analysis. Notched box-plots were constructed for each analysis by both target selection method and probability of kill pair. The notched box-plots give a visual comparison of the means, inter-quartile ranges, and outliers, for different levels of the factors. The Scheffe range test method with a ninety percent confidence level was used for each analysis. The Scheffe range test indicates which of the levels of the factor examined could be placed in homogeneous groups.

TABLE 8  
ANALYSIS OF VARIANCE (ANOVA)  
SIGNIFICANCE LEVELS

Source of Variation	BLUCAS SIM	REDCAS SIM	BLUCAS ALIV	REDCAS ALIV
Covariate				
ENGAGE	0.0000	0.0000	0.0000	0.0000
Main Effect				
TGTSELMETH	0.1686	0.1553	0.0000	0.0000
PKPAIR	0.0000	0.0000	0.0000	0.0000
Two Factor Interactions				
TGTSELMETH- -PKPAIR	0.2676	0.0074	0.5415	0.0077

The first response variables to be examined were the estimated number of expected blue and red casualties obtained from simulations (BLUCASSIM and REDCASSIM) and the aliveness method (BLUCASALIV and REDCASALIV). The significance levels for each source of variation are given in Table 8. Most of the sources of variation were significant (significance values below 0.05 or so). Some of the sources of variation were expected to be significant, especially the number of engagements and the probability of kill pair. Since each engagement in the simulation is an additional opportunity that a player may be killed, the number of casualties for each side tends to increase as the number of engagements increases. In the aliveness method, a partial kill is credited during each engagement, so the number of casualties per side should also increase as the number of engagements increases. The probability

of kill pair was expected to be significant because not all of the simulations used the same probability of kill pair. The PKPAIRs 1, 2, 4, 6, and 8 all use the standard probability of kill pair ( $BPKU = 0.5$  and  $RPKU = 0.2$ ) and should be placed in homogeneous groups when comparing the estimated number of simulated casualties. The PKPAIRs 1, 3, 5, 7, and 9 all adjust the simulation data to the standard probability of kill pair ( $BPKA = 0.5$  and  $RPKA = 0.2$ ) and should be placed in homogeneous groups when comparing the estimated number of casualties using the aliveness method.

TABLE 9  
MULTIPLE RANGE ANALYSIS FOR SIMULATED CASUALTIES  
BY PKPAIR USING 90 PERCENT SCHEFFE METHOD

BLUE CASUALTIES			RED CASUALTIES		
PK PAIR	Average	Homogeneous Groups	PK PAIR	Average	Homogeneous Groups
5	2.475	*	7	5.450	*
9	2.825	*	3	6.275	*
8	3.250	*	4	7.450	*
1	3.400	* *	6	7.575	*
2	3.400	* *	8	8.250	* *
6	3.500	* * *	2	8.275	* *
4	3.650	* * *	1	8.900	*
3	3.875	* *	9	10.200	*
7	3.925	*	5	10.725	*

The multiple range analysis for the estimated number of expected casualties by probability of kill pair (PKPAIR) using simulation is listed in Table 9. The Scheffe method was used to examine forty iterations (ten iterations for each of the four target selection methods) by PKPAIR and to list the PKPAIR by average from lowest to highest. PKPAIRS were assembled into homogenous groups using the Scheffe method with a ninety percent confidence coefficient. PKPAIRs in the same homogeneous group have asterisks (\*) in a common column. For the estimated expected number of casualties by simulation, PKPAIRs 1, 2, 4, 6, and 8 were expected to be in the same

homogeneous group. The expected number of red casualties by simulation had the 1, 2, 4, 6, and 8 PKPAIRs grouped in the middle positions (positions three through seven) of the nine PKPAIRs. The Scheffe method indicates PKPAIRs 1, 2, 6, and 8 were in one homogeneous group and PKPAIRs 1, 2, 4, and 6 were in another. The expected number of blue casualties by simulation also had the 1, 2, 4, 6, and 8 PKPAIRs grouped in the middle positions (positions three through seven). PKPAIRs 2, 4, 6, and 8 were grouped into one homogeneous group and PKPAIRs 1, 2, and 8 were in another.

TABLE 10  
MULTIPLE RANGE ANALYSIS FOR ALIVENESS CASUALTIES  
BY PKPAIR USING 90 PERCENT SCHEFFE METHOD

BLUE CASUALTIES			RED CASUALTIES		
PK PAIR	Average	Homogeneous Groups	PK PAIR	Average	Homogeneous Groups
4	2.769	*	2	5.713	*
8	3.030	* *	7	6.411	* *
9	3.266	* *	6	6.933	* *
7	3.430	* *	3	7.428	*
6	3.597	* * *	4	8.398	*
5	3.645	* *	1	8.566	*
1	3.714	* *	9	8.613	*
2	3.753	* *	8	9.521	*
3	3.810	*	5	10.023	*

The multiple range analysis for the estimated number of expected casualties by probability of kill pair (PKPAIR) using the aliveness method is listed in Table 10. The Scheffe method was used to examine forty iterations for each PKPAIR and to list the PKPAIRs by average from lowest to highest. PKPAIRS 1, 3, 5, 7, and 9 adjusted the simulation data to the standard pair using the aliveness method, so they were expected to be in the same homogeneous group. The expected number of blue casualties using the aliveness method did not assemble the odd PKPAIRs in the middle positions of the nine PKPAIRs. The five odd PKPAIRs were grouped into three different

homogeneous groups. The expected number of red casualties using the aliveness method did not assemble the five odd PKPAIRs in the middle positions of the nine PKPAIRs. The five odd PKPAIRs were grouped into four different homogeneous groups.

TABLE 11  
MULTIPLE RANGE ANALYSIS FOR EXPECTED CASUALTIES  
BY TGTSELMETH USING 90 PERCENT SCHEFFE METHOD

BY SIMULATION

BLUE CASUALTIES			RED CASUALTIES		
TGTSEL METH	Average	Homogeneous Groups	TGTSEL METH	Average	Homogeneous Groups
1	3.211	*	3	7.467	*
4	3.222	*	2	8.056	*
2	3.433	*	4	8.111	*
3	3.600	*	1	8.856	*

BY THE ALIVENESS METHOD

BLUE CASUALTIES			RED CASUALTIES		
TGTSEL METH	Average	Homogeneous Groups	TGTSEL METH	Average	Homogeneous Groups
2	3.219	*	3	6.871	*
1	3.333	*	4	7.203	*
3	3.564	*	1	8.570	*
4	3.667	*	2	9.181	*

The multiple range analysis for the estimated number of expected casualties by target selection method (TGTSELMETH) is listed in Table 11. The Scheffe method was used to examine ninety iterations (ten iterations for each of the nine probability of kill pairs) for each TGTSELMETH and to list the TGTSELMETH by expected casualty average from lowest to highest. The TGTSELMETH were assembled into groups using the Scheffe method with a ninety percent confidence interval. There was a different ordering of the TGTSELMETH for each expected number of red and blue casualties by each method, simulation and aliveness, as shown in Table 11.

The unexpected results in the multiple range analysis by both PKPAIR and TGTSELMETH and the high levels of significance with the two-factor interactions as a source of variation prompted further investigation into the interaction of PKPAIR and TGTSELMETH. Multiple x-y plots were constructed using STATGRAPHICS. The mean number of red casualties per ten iterations were plotted against the PKPAIRs. If there is little or no interaction, the lines connecting the means utilizing the same target selection method should be parallel, to within the variability of the sample means. Multiple crossings of the connecting lines indicates a strong interaction. Both plots of red casualties by simulation and the aliveness method contained multiple crossings. The plots contained a lot of "noise" because not all of the probability of kill pairs were comparable. PKPAIRs 3, 5, 7, and 9 were eliminated from the plot of mean expected red casualties by simulation. The plot showed much interaction between TGTSELMETH and PKPAIR. The target selection methods displayed more variability than expected, considering that each data point is an average of ten iterations. The first target selection method (random selection) appears to have the smallest range. PKPAIRs 2, 4, 6, and 8 were eliminated from the plot of expected red casualties by the aliveness method. The resulting plot showed little interaction between TGTSELMETH and PKPAIR. The target selection methods appear to act more in concert with each other, but with each at its own level. This indicates that the method of target selection has an effect on the aliveness adjustments. The direction and magnitude of the aliveness adjustments appear to make a difference in how well the aliveness method compares with "ground truth." The mean number of red casualties using the aliveness method change in the same direction for all four target selection methods for each probability of kill pair. The two plots are shown in Appendix F.

To gain more insight into the ability of the aliveness adjustments to give values comparable with "ground truth" and to utilize all of the available data, the differences between the expected number of estimated red and blue casualties by simulation and the aliveness methods were used as response variables. For each of the nine probability of kill pairs (PKPAIRs), the estimated number of expected casualties using the aliveness method was subtracted from the corresponding estimated number of casualties by simulation. For example, the aliveness method was used with PKPAIR 2 to adjust from the simulated data using the standard probability of kill pair to BPKA = 0.25 and RPKA = 0.20. That same probability of kill pair (BPKU = 0.25 and

RPKU = 0.20) was used for the simulation with PKPAIR 3 . The estimated number of expected casualties using the aliveness method with PKPAIR 2 can then be subtracted from the estimated number of expected casualties using the PKPAIR 3 simulation since the probabilities of kill are comparable. All nine PKPAIRs listed in Table 7 were utilized in a similar fashion. Since the estimated number of expected casualties using the aliveness method was subtracted from the estimated number of expected casualties using the corresponding simulation, a negative difference means that the estimated number of expected casualties by the aliveness method was greater than the simulation. A positive difference means that the estimated number of expected casualties by simulation was greater than that using the aliveness method.

TABLE 12  
ANALYSIS OF VARIANCE (ANOVA)  
SIGNIFICANCE LEVELS

Source of Variation	BLUEDIFF	REDDIFF
Covariate		
ENGAGE	0.0000	0.0000
Main Effect		
TGTSELMETH	0.0164	0.0000
PKPAIR	0.2875	0.0000
Two Factor Interactions		
TGTSELMETH- -PKPAIR	0.0700	0.0939

Differences in the estimated numbers of blue and red casualties (BLUEDIFF and REDDIFF) between a simulation and the corresponding aliveness method were examined. The significance levels for each source of variation for the analysis of variance procedure are given in Table 12. Five of the eight sources of variation had significance levels less than 0.05 and were, therefore, significant. As explained previously, the number of engagements was expected to be a significant source of variation, and it was for both BLUEDIFF and REDDIFF. The target selection method (TGTSELMETH) was also significant for both BLUEDIFF and REDDIFF,

but the probability of kill pair (PKPAIR) was only significant for REDDIFF. The two-factor interactions were marginally insignificant. Further investigation of the main effects included construction of notched box plots, which are displayed in Appendix G.

TABLE 13  
MULTIPLE RANGE ANALYSIS FOR DIFFERENCES IN CASUALTIES,  
BY PKPAIR, USING 90 PERCENT SCHEFFE METHOD

DIFFERENCE IN BLUE CASUALTIES			DIFFERENCE IN RED CASUALTIES		
PK PAIR	Average	Homogeneous Groups	PK PAIR	Average	Homogeneous Groups
2	-0.410	*	4	-2.571	*
1	-0.314	*	7	-1.483	* *
5	-0.294	* *	8	-0.363	* *
9	-0.205	* *	1	0.334	*
8	-0.016	* *	3	0.563	*
4	0.005	* *	9	0.679	*
6	0.070	* *	2	0.847	* *
3	0.122	* *	6	1.164	* *
7	0.328	*	5	2.327	*

The multiple range analysis for the difference in the estimated number of expected casualties, by probability of kill pair (PKPAIR), is listed in Table 13. The Scheffe method was used to examine forty differences (ten differences for each of the target selection methods) and to list the PKPAIRs by average difference from lowest to highest. It is evident why the PKPAIR as a source of variation was insignificant for the difference in expected blue casualties (BLUEDIFF). All nine PKPAIRs are contained in only two homogeneous groups using a ninety percent Scheffe confidence coefficient and six of the PKPAIRs are contained in both groups. The difference in expected red casualties (REDDIFF) is significantly dependent on which PKPAIR is used. The nine PKPAIRs are assembled in four homogeneous groups. The notched box plot in Figure G.2 gives a visual comparison of the difference in expected red casualties as a function of PKPAIR. The box plot strongly suggests that the direction

and magnitude of the aliveness adjustments make a difference in how well the aliveness method compares to the simulation results (indicated by the sign and magnitude of REDDIFF).

TABLE 14  
MULTIPLE RANGE ANALYSIS FOR DIFFERENCES IN CASUALTIES,  
BY TGTSELMETH, USING 90 PERCENT SCHEFFE METHOD

DIFFERENCE IN BLUE CASUALTIES			DIFFERENCE IN RED CASUALTIES		
TGTSEL METH	Average	Homogeneous Groups	TGTSEL METH	Average	Homogeneous Groups
4	-0.445	*	2	-1.125	*
1	-0.122	*	1	0.286	*
3	0.036	* *	3	0.596	*
2	0.214	*	4	0.908	*

The multiple range analysis for the difference in the estimated number of expected casualties, by target selection method (TGTSELMETH), is listed in Table 14. The Scheffe method was used to examine ninety differences (ten differences for each of the nine comparable probability of kill pairs) for each TGTSELMETH and to list the TGTSELMETH by average difference from lowest to highest. As a source of variation on the analysis of variance, TGTSELMETH was significant for both BLUEDIFF and REDDIFF. The nature of the differences is displayed in Table 14 and Figures G.3 and G.4. There is strong evidence that the target selection method significantly affects the comparison of the aliveness method and simulation results. It is interesting to note that target selection methods 2 and 4 were on opposite ends of the scale for BLUEDIFF and REDDIFF. One can infer that while using TGTSELMETH 2, selecting as the target the opposing player with the greatest aliveness, the aliveness adjustments favor the blue side. The positive BLUEDIFF average indicates that the aliveness method does not estimate as many expected blue casualties as does the simulation. The negative REDDIFF average indicates that the aliveness method estimates a higher number of expected red casualties than the simulation. The

opposite effect is the case with TGTSELMETH 4, selecting as the target the opposing player most dangerous to the firer. The aliveness adjustments using TGTSELMETH 4 favor the red side. The aliveness adjustments using TGTSELMETH 1, selecting the target by random number draw, marginally favor the red side. This blue-to-red change over is probably evident because the simulations involve a duel between blue and red players.

The two-factor interactions, which were marginally insignificant, were examined to gain additional insight into the behavior of the different target selection methods. Multiple x-y plots of the differences in the estimated expected number of casualties by target selection method (TGTSELMETH) as functions of probability of kill pair (PKPAIR) were constructed and are displayed in Appendix G. The aliveness method increases the blue probability of kill (BPK) with PKPAIRs 2 and 9 and decreases the BPK with PKPAIRs 3 and 8. The aliveness method increases the red probability of kill (RPK) with PKPAIRs 4 and 7 and decreases the RPK with PKPAIRs 5 and 6. The only noteworthy observation about the interaction plot of the mean differences of blue casualties (Figure G.5) is that TGTSELMETH 2 tends to follow a pattern. Using TGTSELMETH 2, the aliveness method underestimates the number of expected number of blue casualties when the BPK decreases or the RPK increases and overestimates the number of expected blue casualties when the BPK increases or the RPK decreases. There are several interesting observations about the interaction plot of the mean differences of the red casualties (Figure G.6). The aliveness method overestimates the expected number of red casualties utilizing all four target selection methods whenever the RPK increases. The aliveness method overestimates the expected number of red casualties for TGTSELMETH 2 whenever the BPK or RPK increases. The aliveness method underestimates the expected number of red casualties for TGTSELMETH 4 whenever the BPK or RPK decreases.

Plots of the residuals from BLUEDIFF and REDDIFF against the predicted values from the analysis of variance were constructed. A careful examination of both plots revealed that the assumptions of normality and homoscedasticity, required for the classical analysis of variance, appear to be tenable.

In an attempt to find a single measure of the accuracy of the aliveness method, another variable was created. The new variable is the square root of the sum of squared differences of the number of expected casualties between simulation and aliveness methods (SQRTSSD). The differences in the number of expected casualties

between methods were already calculated (BLUEDIFF and REDDIFF). Since the red force was three times as large as the blue force and therefore the expected number of red casualties are very roughly three times as variable as the expected number of blue casualties (zero to twelve versus zero to four), the differences in the number of expected blue casualties were weighted by a factor of three. In mathematical form, the SQRSSD was calculated for each pair of differences in the following manner:

$$\text{SQRSSD} = \text{SQRT}((3*(\text{BLUEDIFF})^2) + (\text{REDDIFF})^2) . \quad (3.1)$$

An analysis of variance was performed on the square root of the sum of squared differences (SQRSSD) data. Every source of variation was insignificant, although the probability of kill pair (PKPAIR) was only marginally insignificant. Examination of the SQRSSD residuals plotted against predicted values indicated a heterogeneity of variance. A logarithmic transformation was applied to the SQRSSD data and another analysis of variance was performed. Every source of variation, except for PKPAIRs was insignificant. The residuals of the log of SQRSSD were plotted against the predicted values (see Figure H.1) and a more suitable plot was produced. The significance levels of the PKPAIR and TGTSELMETH were .0009 and .7459, respectively.

The multiple range analysis for the square root of the sum of squared differences (SQRSSD), by probability of kill pair (PKPAIR) and target selection method (TGTSELMETH), is listed in Table 15. The Scheffe method was used to examine forty data points per PKPAIR and ninety data points per TGTSELMETH and to list PKPAIR and TGTSELMETH by average from highest to lowest. The nine PKPAIRs are contained in three homogeneous groups using a ninety percent Scheffe confidence coefficient. It is evident that PKPAIR is significant, although there is no discernable pattern. Figure H.2 gives a visual comparison of the SQRSSD as a function of PKPAIR. It is interesting to note that TGTSELMETH is quite insignificant for SQRSSD. Figure H.3 displays how similar each of the box plots are to each other. Perhaps the difference in the number of expected blue casualties (BLUEDIFF) and the difference in the number of expected red casualties (REDDIFF) combined in this manner in SQRSSD compensate for each other. It is also believed that this may be the case for the covariate, number of engagements (ENGAGE). In all previous analysis of variance, ENGAGE was highly significant (significance level of 0.0000), but its significance level in the analysis of variance for SQRSSD was 0.6897.

TABLE 15  
MULTIPLE RANGE ANALYSIS FOR SQR TSSD  
USING THE 90 PERCENT SCHEFFE METHOD

PK PAIR	BY PKPAIR		BY TGTSELMETH		
	Average	Homogeneous Groups	TGTSEL METH	Average	Homogeneous Groups
1	3.265	*	1	4.098	*
3	3.642	* *	4	4.328	*
9	4.169	* * *	3	4.489	*
7	4.213	* * *	2	4.530	*
8	4.627	* *			
4	4.707	* *			
6	4.729	* *			
5	4.816	* *			
2	5.082	*			

#### IV. EXPERIMENTAL RESULTS SUMMARY

The accuracy of aliveness adjustments is sensitive to changes in the battle parameters. Two experiments were designed to examine the sensitivity of the accuracy of the aliveness method to changes in the probability of kill and the target selection method. The aliveness adjustments were examined over fourteen probability of kill pairs in experiment one. In experiment two, the aliveness adjustments were examined over nine probability of kill pairs and four target selection methods.

The accuracy of aliveness adjustments is sensitive to changes in the probability of kill. It was discovered in experiment one that the amount of change from used probability of kill (PKU) to actual probability of kill (PKA) and to which side that change was applied affected the quality of estimation of the mean number of red and blue casualties (Figures 3.1 and 3.2). The effect of change in the probability of kill was further examined in experiment two. The probability of kill pair (PKPAIR) variable was significant in six of the seven analyses of variance (Tables 8 and 12). The sensitivity of the accuracy of the aliveness adjustments to the degree of changes in the probability of kill was very evident in the analysis of variance on the differences between the expected number of casualties by aliveness and the corresponding simulation. The box plots (Figures G.1 and G.2) of these differences in casualties give a visual summarization of the effect of the probability of kill pair on the accuracy of the aliveness adjustments. It appears that the PKPAIR has more impact as the number of players increases. PKPAIR was the only variable that was significant in the analysis of variance on the square root of the sum of the squared differences of the number of expected casualties (SQRTSSD). The ninety percent Scheffe multiple range analysis (Table 15) and the notched box plot (Figure H.2) give visual indications of how SQRTSSD varies with PKPAIR.

The accuracy of aliveness adjustments is sensitive to which target selection method is being used by the firer. The target selection method (TGTSSELMETH) variable was significant in four of the seven analyses of variance (Tables 8 and 12). The TGTSSELMETH appears to be more significant with a larger number of players. TGTSSELMETH was not significant in the analysis of variance with the number of expected blue casualties by either the aliveness method or simulation, but was highly

significant in the analysis of variance with the number of expected red casualties by either method. The plots of the two-factor interaction of the mean number of red casualties by probability of kill pair (PKPAIR) utilizing all four TGTSELMETH were interesting. The two factor interaction on mean red simulation casualties (Figure F.1) showed significant differences between the TGTSELMETHs, although a pattern in the differences is not obvious. The two-factor interaction on mean red aliveness casualties (Figure F.2) was significant, and these differences follow a pattern. Each TGTSELMETH followed a similar pattern, although for different levels of mean number of red casualties. TGTSELMETH was significant in the analysis of variance on the differences in the number of expected casualties. The notched box plots (Figures G.3 and G.4) and the multiple range analysis using the ninety percent Scheffe method (Table 14) give visual summaries of the sensitivity of the aliveness adjustments to TGTSELMETH. The two-factor interactions of TGTSELMETH and PKPAIR uncovered several tendencies of the TGTSELMETH. From the interaction of TGTSELMETH and PKPAIR on the mean difference in blue casualties (Figure G.5), it was apparent that TGTSELMETH 2, selecting the target with the highest aliveness value, underestimates the number of expected blue casualties whenever the blue probability of kill (BPK) decreased or the red probability of kill (RPK) increased. From the interaction of TGTSELMETH and PKPAIR on the mean difference in red casualties (Figure G.6), it is apparent that the aliveness method overestimates the number of red casualties for TGTSELMETH 2 whenever the BPK or RPK increases and underestimates the number of red casualties for TGTSELMETH 4, selecting the target that has the highest probability of kill (PKU) against the firer, whenever the BPK or RPK decrease. In the analysis of variance on the single measure of the accuracy of the aliveness adjustments (SQRTSSD), the TGTSELMETH was insignificant. Since TGTSELMETH was significant for the analyses of variance on the components of SQRTSSD, but not for the analysis of variance on SQRTSSD itself, some type of compensatory effect appears to be occurring.

## V. CONCLUSIONS AND RECOMMENDATIONS

### A. CONCLUSIONS

The aliveness method provides estimates of expected casualties with remarkable accuracy. Dr. Bryson and Dr. Russell demonstrated great insight in developing the aliveness method. Using the aliveness method, analysts can reap benefits from field tests that had used incorrect probabilities of kill. The aliveness technique does give an occasional wild aliveness value. In two instances out of over six hundred iterations, the aliveness method computed extremely high (over forty) aliveness values and credited kills. For our analysis, those instances were considered outliers and were replaced by values from two new iterations. The aliveness method appears to reduce variance relative to estimation by observed casualties (Table 6 and Figures B.2 and B.4), especially as the number of players increase. The aliveness method appears to be sensitive to the difference between the used probability of kill (PKU) and the actual probability of kill (PKA), and target selection method. The aliveness method can be a time and money saving tool for experimenters. Experimenters must be aware of possible bias when designing experiments or when utilizing the aliveness method.

One measure that can be used to assess bias is the difference in the estimated number of casualties between a simulation and corresponding estimates using the aliveness method. Using this measure, it was found that the aliveness method is affected by the amount, direction and force to which a change in the probability of kill pertains. The aliveness method is most biased when estimating the number of casualties of the most numerous force while adjusting the probability of kill of that force. In experiment two, the aliveness method overestimated the number of red casualties by over thirty-four percent (PKPAIR 4, RPK changes from 0.1 to 0.2) and underestimated the number of red casualties by over twenty-one percent (PKPAIR 5, RPK changes from 0.2 to 0.1). The aliveness method is least biased while adjusting the probability of kill of the smaller, more potent force.

The same measure, the difference in the estimated number of casualties between a simulation and the corresponding aliveness method, was used to assess the relationship between bias and target selection method (TGTSELMETH). All four examined TGTSELMETHs affected the accuracy of the aliveness adjustments to different

degrees. The aliveness adjustments with TGTSELMETH 2, selecting as the target the player with the greatest aliveness, are biased toward the smaller more potent (higher probability of kill) force. The aliveness adjustments with TGTSELMETH 4, selecting as the target the player most dangerous to the firer, are biased toward the larger, less potent force. The aliveness adjustments with TGTSELMETH 1, selecting a target at random, were also found to slightly favor the larger less potent force. The aliveness adjustments with TGTSELMETH 3, selecting as the target the player most vulnerable to the firer, underestimated the number of expected casualties in most cases.

## **B. RECOMMENDATIONS**

The following recommendations are made for further investigation and study:

1. A theoretical basis for the aliveness concept should be developed. The development of a theoretical basis for the aliveness concept may help explain some of the bias exhibited by the aliveness method.
2. If it is to be used further, the simulation program should be modified to reflect a more realistic method of choosing a firing side. Presently the aliveness program uses a random number draw and each side has a 0.50 probability of the firer coming from that side. Perhaps the selection of firing side should be based on some factor such as the ratio of surviving players per side.
3. Since a partial kill is credited for each engagement in the aliveness method, the accuracy of the aliveness adjustments as a function of the number of engagements should be examined.
4. The force structures (the number and the ratio of red players to blue players) should be varied to test the significance of large force changes in the accuracy of the aliveness method.
5. If a battle is designed to go longer, say when the probabilities of kill are small, ammunition constraints should be incorporated into the program to limit the number of times an individual player may fire. Possibly the battle could be stopped when one of the forces reaches some set fraction of its original strength.
6. The amounts and directions of changes to the probabilities of kill should be further examined. The effects of a wider range of changes to a probability of kill and of changes to both probabilities of kill simultaneously could be used as a basis to attempt to establish a relationship between the amount and direction of change in the probability of kill and the bias of the aliveness adjustments.

## APPENDIX A

### MODIFICATION OF THE ALIVENESS PROGRAM

This appendix contains the modification of the original aliveness program ("ALIVE.BAS", created by Dr. Carl Russell) that was used in the first experiment and for the random target selection method (TGTSELMETH 1) in the second experiment.

```

1000 REM***
1010 REM*** BASICA PROGRAM, "ALIVE1.BAS" WHICH DOES ALIVENESS ANALYSES
      ON SIMULATED DATA IS A MODIFIED VERSION OF "ALIVE.BAS"
      ORIGINALLY CREATED BY DR. CARL RUSSELL
1020 REM***
1030 REM*** INITIALIZATION ROUTINES
1040 REM***
1050 CLS
1060 LPRINT CHR$(27); "A"; "07"
1070 WIDTH "LPT1:"; 132
1075 OPEN "B:ALIV" FOR APPEND AS #1
1080 PKGOOF=.25 : BLUE=12 : PKB=.5 : BS=1 : BPKA=.6 : BPKU=.2 :
      BJIT=0! : RED=12 : PKR=.5 : RS=1 : RPKA=.6 : RPKU=.8 :
      RJIT=0! : ITER=3 : MTH=3 : POPT=2
1085 ITER=10
1090 XS=TIMES: SEED=3600*VAL(MID$(XS,1,2))+60*VAL(MID$(XS,4,2))+
      VAL(MID$(XS,7,2))
1100 INPUT "Do you want to print the DESCRIPTION OF ANALYSES (Y/N)";
      YESORNOS
1110 IF YESORNOS="y" OR YESORNOS="Y" THEN GOSUB 2760
1120 YESORNOS=""
1130 INPUT "Do you want to print INTERIM SUMMARIES after every page
      (Y/N)"; YESORNOS
1140 IF YESORNOS="n" OR YESORNOS="N" THEN SMRY=0 ELSE SMRY=1
1150 YESORNOS=""
1160 INPUT "Do you want to specify particular PkA's and PkU's (Y/N)";
      YESORNOS
1170 IF YESORNOS="y" OR YESORNOS="Y" THEN GOSUB 2210: GOTO 1250
1180 INPUT "Do you want to generate PKs using the original method (Y/N)";
      YESORNOS
1190 IF YESORNOS<>"y" AND YESORNOS<>"Y" GOTO 1250
1200 POPT=0
1210 INPUT "Number of Blue Players, Average Pk for Blue Against Red,
      Blue Pk Spread"; BLUE, PKB, BS
1220 INPUT "Number of Red Players, Average Pk for Red Against Blue,
      Red Pk Spread"; RED, PKR, RS
1230 INPUT "Probability of RTCA Goof, Number of Iterations"; PKGOOF,
      ITER
1240 INPUT "Random Number Seed"; SEED
1250 YESORNOS=""
1260 TOTAL=RED+BLUE: RANDOMIZE SEED
1270 DIM PK(TOTAL,TOTAL,2), A(TOTAL,MTH), K(TOTAL,TOTAL,MTH),
      PLAYER(TOTAL), STS(TOTAL), KDCNT(MTH), KSCNT(MTH), ALTOT(MTH),
      ENG(TOTAL,1), DENA(TOTAL,MTH), RATIO(TOTAL,MTH),
      EXPA(TOTAL,MTH), RAT(MTH), SPKA(TOTAL,1), SPKU(TOTAL,1),
      APKA(1), APKU(1)
1280 REM***
1290 REM*** COMPUTE AND PRINT PK MATRIX
1300 REM***
1310 GOSUB 2300
1320 REM***
1330 REM*** INITIALIZE ENGAGEMENT FOR-NEXT LOOP
1340 REM***
1350 FOR ITERNUM=1 TO ITER: RNUM=RED: BNUM=BLUE: RPOS=BLUE+1

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1360 E=0: FOR L=0 TO TOTAL: PLAYER(L)=L: STS(L)=1: FOR FT=0 TO 1:
      ENG(L,FT)=0: SPKA(L,FT)=0: SPKU(L,FT)=0: NEXT FT: FOR K=1 TO MTH:
      A(L,K)=1: DENA(L,K)=1: EXPA(L,K)=1: RATIO(L,K)=0:
      FOR M=0 TO TOTAL: K(L,M,K)=0: NEXT M,K,L
1390 TOT=RNUM+BNUM: E=E+1
1400 REM***
1410 REM*** CHOOSE FIRER AND TARGET
1420 REM***
1430 IF RND<.5 THEN I1=INT(RND*BNUM+1) ELSE I1=INT(RND*RNUM+RPOS)
1440 I=PLAYER(I1)
1450 IF I1>BNUM THEN J1=INT(RND*BNUM+1) ELSE J1=INT(RND*RNUM+RPOS)
1460 J=PLAYER(J1)
1470 ENG(I,0)=ENG(I,0)+1: ENG(J,1)=ENG(J,1)+1
1480 REM***
1490 REM*** FIND PKs AND ASSESS REAL TIME KILLS
1500 REM***
1510 RTKILL=0
1520 PKA=PK(I,J,1)
1530 PKU=PK(I,J,2)
1540 IF POPT<2 GOTO 1570
1550 IF J>BNUM THEN JIT=BJIT ELSE JIT=RJIT
1560 PJIT=PKA: GOSUB 2690: PKA=PJIT: PJIT=PKU: GOSUB 2690: PKU=PJIT
1570 IF RND<PKGOOF THEN PKU=0: RTKILL=-1
1580 SPKA(I,0)=SPKA(I,0)+PKA: SPKA(J,1)=SPKA(J,1)+PKA
1590 SPKU(I,0)=SPKU(I,0)+PKU: SPKU(J,1)=SPKU(J,1)+PKU
1600 IF RND>PKU GOTO 1650 ELSE RTKILL=1: STS(J)=0:
      IF AZ>BNUM THEN RNUM=RNUM-1 ELSE RPOS=RPOS-1: BNUM=BNUM-1
1610 IF AZ>TOT THEN FOR L=AZ TO TOT-1: PLAYER(L)=PLAYER(L+1): NEXT L
1620 REM***
1630 REM*** CREDIT KILLS AND DECREMENT ALIVENESS
1640 REM***
1650 FOR L=1 TO MTH: OLDKILL=K(I,J,L): DELTA=A(I,L)*A(J,L)*PKA
1660 OLDAJ=A(J,L)
1670 IF PKU<1 AND L=3 THEN A(J,3)=A(J,3)*(1-PKA)**
      A(I,3)/(1-PKU): DELTA=OLDAJ*(1-(1-PKA)**A(I,3)
1680 IF PKU<1 AND L=2 THEN A(J,2)=A(J,2)*(1-PKA)**A(I,2)
1690 IF PKU<1 AND L=1 THEN A(J,1)=A(J,1)*(1-A(I,1)*PKA)
1700 IF A(J,1)<0 THEN A(J,1)=0: DELTA=OLDAJ: RTKILL=RTKILL-
1710 REM *** IF RTKILL>=1 THEN A(J,1)=0
1720 K(I,J,L)=K(I,J,L)+DELTA: K(O,J,L)=K(O,J,L)+DELTA
      K(I,O,L)=K(I,O,L)+DELTA
1730 DENA(J,L)=DENA(J,L)*(1-PKU): EXPAJ=DENA(J,L)*A(I,L)
      EXPA(J,L)=EXPAJ
1740 KLO=K(O,J,L): KL=K(I,J,L): SUMEK=EXPAJ+KLO
1750 IF SUMEK>.00001 THEN RATIO(J,L)=KLO/(EXPAJ+KLO) ELSE RATIO(J,L)=0
1760 REM***
1770 REM*** PRINT ENGAGEMENT RESULTS
1780 REM***
1850 NEXT L
1860 IF RNUM>0 AND BNUM>0 THEN GOSUB 2160: GOTO 1390
1870 PRINT: PRINT USING "End of Iteration###: ## Blue and
      ## Red Players Remain.": ITERNUM; BNUM; RNUM
1880 REM*** LPRINT USING "End of Iteration###": ITERNUM
1890 REM***
1900 REM*** PRINT ITERATION SUMMARY
1910 REM***
1920 GOSUB 3150
1930 REM *** VS=INKEYS: IF VS="" GOTO 1900
1940 CLS: NEXT ITERNUM
1945 CLOSE #1
1950 END
1960 REM *****
      *****
1970 REM***
1980 REM*** SUBROUTINE TO PRINT PAGE HEADERS
1990 REM***
2000 REM*** PAGENO=PAGENO+1: LPRINT CHR$(12);: LPRINT USING "Aliveness
      Analyses of Simulated Data Done on          at
      Page###; DATE$; X$; PAGENO: LPRINT: LINO=3
2010 RETURN

```

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2020 REM***
2030 REM*** SUBROUTINE TO PRINT ENGAGEMENT LIST HEADER
2040 REM***
2050 REM***
2090 RETURN
2100 REM***
2110 REM*** SUBROUTINE TO PRINT ITERATION SUMMARY HEADER
2120 REM***
2130 GOSUB 2000
2140 PRINT #1, USING "Summary of Iteration###: ## Blue and ##
      Red Players Remain after### Engagements."; ITERNUM; BNUM; RNUM; E
2150 RETURN
2160 REM***
2170 REM*** SUBROUTINE FOR PAGING
2180 REM***
2190 IF LINO>55-MTH-1 THEN IF SMRY=1 THEN GOSUB 3150: GOSUB 2050:
      ELSE GOSUB 2050 ELSE LINO=LINO+MTH+1
2200 RETURN
2210 REM***
2220 REM*** SUBROUTINE TO INPUT PKA'S AND PKU'S
2230 REM***
2240 INPUT "Do you want jitter Pk's only once (Y/N)"; YESORNOS
2250 IF YESORNOS="y" OR YESORNOS="Y" THEN POPT=1 ELSE POPT=2
2260 INPUT "Number of Blue Players, Number of Red Players"; BLUE, RED
2270 INPUT "PKA, PKU, Jitter for Blue firer and Red Target";
      BPKA, BPKU, BJIT
2280 INPUT "PKA, PKU, Jitter for Red firer and Blue Target";
      RPKA, RPKU, RJIT
2290 RETURN
2300 REM***
2310 REM*** SUBROUTINE TO DEFINE AND PRINT PK MATRIX
2320 REM***
2330 FOR L=1 TO TOTAL: FOR M=1 TO TOTAL
2340 IF (L>BLUE AND M>BLUE) OR (L<BLUE+1 AND M<BLUE+1) GOTO 2450
2350 ON POPT+1 GOTO 2400, 2380, 2360
2360 IF L<BLUE+1 AND M>BLUE THEN PK(L,M,1)=BPKA: PK(L,M,2)=BPKU:
      ELSE PK(L,M,1)=RPKA: PK(L,M,2)=RPKU
2370 GOTO 2450
2380 IF L<BLUE+1 AND M>BLUE THEN PJIT=BPKA: JIT=BJIT: GOSUB 2690:
      PK(L,M,1)=PJIT: PJIT=BPKU: GOSUB 2690: PK(L,M,2)=PJIT: ELSE
      PJIT=RPKA: JIT=RJIT: GOSUB 2690: PK(L,M,2)=PJIT
2390 GOTO 2450
2400 IF L<BLUE+1 AND M>BLUE THEN PKX=PKB: SPD=BS: ELSE PKX=PKR: SPD=RS
2410 IF PKX<1-PKX THEN PK1=PKX+PKX*(RND-.5)*2*SPD ELSE PK1=PKX+
      (1-PKX)*(RND-.5)*2*SPD
2420 IF PK1<1-PK1 THEN PK2=PK1+PK1*(RND-.5)*2*SPD ELSE PK2=PK1+
      (1-PK1)*(RND-.5)*2*SPD
2430 IF PK1>.9899999 THEN PK(L,M,1)=.9899999 ELSE IF PK1<.01
      THEN PK(L,M,1)=.01 ELSE PK(L,M,1)=PK1
2440 IF PK2>.9899999 THEN PK(L,M,2)=.9899999 ELSE IF PK2<.01
      THEN PK(L,M,2)=.01 ELSE PK(L,M,2)=PK2
2450 NEXT M, L
2460 GOSUB 2000
2470 IF POPT>0 GOTO 2510
2480 PRINT #1, "PARAMETERS ARE: BLUE PKB BS RED PKR
      RS SEED PKGOOF POPT"
2490 PRINT #1, USING "
      .## .## ##### .## "### .## ###
      PKGOOF; POPT
2500 GOTO 2530
2510 PRINT #1, "PARAMETERS ARE: BLUE BPKA BPKU BJIT RED
      RPKA RPKU RJIT SEED PKGOOF POPT"
2520 PRINT #1, USING "
      ### .## .## .## ##### .## "###
      RED; RPKA; RPKU; RJIT; SEED; PKGOOF; POPT
2530 PRINT #1, "PRINT #1, "Pk Table "
2540 PRINT #1, "PRINT #1, "Blue Firer Against Red Target"
2550 PRINT #1, "PRINT #1, "TARGET ";: FOR M=BLUE+1 TO TOTAL:
      PRINT #1, USING " ## "; M,: NEXT M: PRINT #1, " AVERAGE"
2560 PRINT #1, " FIPER ";: FOR M=BLUE+1 TO TOTAL+1:

```

```

PRINT #1, " PKA PKU"; NEXT M: PRINT #1,
2570 FOR L=1 TO BLUE: PRINT #1, USING " ## "; L;: FOR M=BLUE+1
TO TOTAL: PRINT #1, USING " .## .##"; PK(L,M,1); PK(L,M,2);:
PK(L,0,1)=PK(L,0,1)+PK(L,M,1): PK(L,0,2)=PK(L,0,2)+PK(L,M,2):
PK(0,M,1)=PK(0,M,1)+PK(L,M,1): PK(0,M,2)=PK(0,M,2)+PK(L,M,2): NEXT M
2580 PRINT #1, USING " .## .##"; PK(L,0,1)/RED; PK(L,0,2)/RED:
PK(0,0,1)=PK(0,0,1)+PK(L,0,1): PK(0,0,2)=PK(0,0,2)+PK(L,0,2): NEXT L
2590 PRINT #1, "AVERAGE";: FOR M=BLUE+1 TO TOTAL: PRINT #1, USING "
.## .##"; PK(0,M,1)/BLUE; PK(0,M,2)/BLUE;: NEXT M
2600 PRINT #1, USING " .## .##"; PK(0,0,1)/(RED*BLUE);
PK(0,0,2)/(RED*BLUE): PK(0,0,1)=0: PK(0,0,2)=0
2610 PRINT #1, "Red Firer Against Blue Target"
2620 PRINT #1, "TARGET";: FOR M=1 TO BLUE:
PRINT #1, USING " ## "; M;: NEXT M: PRINT #1, " AVERAGE"
2630 PRINT #1, " FIRER ";: FOR M=1 TO BLUE+1: PRINT #1, " PKA PKU";:
NEXT M: PRINT #1,
2640 FOR L=BLUE+1 TO TOTAL: PRINT #1, USING " ## "; L;:
FOR M=1 TO BLUE: PRINT #1, USING " .## .##"; PK(L,M,1); PK(L,M,2);:
PK(L,0,1)=PK(L,0,1)+PK(L,M,1): PK(L,0,2)=PK(L,0,2)+PK(L,M,2):
PK(0,M,1)=PK(0,M,1)+PK(L,M,1): PK(0,M,2)=PK(0,M,2)+PK(L,M,2): NEXT M
2641 NEXT M
2650 PRINT #1, USING " 0## 0##"; PK(L,0,1)/BLUE; PK(L,0,2)/BLUE:
PK(0,0,1)=PK(0,0,1)+PK(L,0,1): PK(0,0,2)=PK(0,0,2)+PK(L,0,2)
2651 NEXT L
2660 PRINT #1, "AVERAGE";: FOR M=1 TO BLUE: PRINT #1, USING " .##
.##"; PK(0,M,1)/RED; PK(0,M,2)/RED;: NEXT M
2670 PRINT #1, USING " .## .##"; PK(0,0,1)/(RED*BLUE);
PK(0,0,2)/(RED*BLUE)
2680 RETURN
2690 REM***
2700 REM***SUBROUTINE WHICH ADDS JITTER TO PK'S
2710 REM***
2720 RNJIT=2*JIT*(RND-.5)
2730 IF RNJIT>0 THEN PJIT=PJIT+RNJIT*(1-PJIT) ELSE PJIT=PJIT*(1+RNJIT)
2740 IF PJIT>.99 THEN PJIT=.99 ELSE IF PJIT<.01 THEN PJIT=.01
2750 RETURN
2760 REM***
2770 REM***SUBROUTINE WHICH WRITES DESCRIPTION OF METHODS
2780 REM***
2790 LPRINT CHR$(12): LPRINT: LPRINT: LPRINT "
DESCRIPTION OF ALIVENESS ANALYSIS METHODS ANDPRINT OUT"
2800 LPRINT: LPRINT " All three methods start with A(I,J)=1."
2810 LPRINT: LPRINT " For an engagement where player I
fires at player J with probability of kill PKA where PKU was used
for RTCA:"
2820 LPRINT " Method 1 credits A(J)*A(I)*PKA=
A(J)*(1-(1-A(I)*PKA)) kills by I against J"
2830 LPRINT " and adjusts A(J) by the
factor (1-A(I)*PKA)/(1-PKU)."
2840 LPRINT " (NOTE: If (1-A(I)*PKA) is negative, only A(J) kills are credited,
and A(J) is reduced to zero.)"
2850 REM *** LPRINT " (FURTHER NOTE: In case of a simulated real time kill, A(J) is also
reduced to zero for Method 1.)"
2860 LPRINT " Method 2 credits A(J)*A(I)*PKA=
A(J)*(1-(1-A(I)*PKA)) kills by I against J"
2870 LPRINT " and adjusts A(J) by the
factor (1-PKA)/(1-PKU)."
2880 LPRINT " Method 3 credits A(J)*(1-(1-PKA))*
A(I)) kills by I against J"
2890 LPRINT " and adjusts A(J) by the
factor (1-PKA)**A(I)/(1-PKU)."
2900 LPRINT: LPRINT " On the Engagement List:"
2910 LPRINT " E is the engagement number,"
2920 LPRINT " I is the firer ID and J is the
target ID (low IDs indicate Blue, high IDs indicate Red),"
2930 LPRINT " PKA is the actual PK and PKU is the
value used in simulated RTCA,"
2940 LPRINT " A(I), A(J), and NEWAJ are the

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    aliveness values,"
2950 LPRINT "          OLDKL is the old cumulative
      credited kills by I against J,"
2960 LPRINT "          DELTA is the change in cumulative
      credited kills by I against J,"
2970 LPRINT "          NEWKL is the new cumulative
      credited kills by I against J,"
2980 LPRINT "          TOTKL is the overall cumulative
      credited kills by any player against J,"
2990 LPRINT "          DENAJ is the denominator of NEWAJ,
      the probability that J survives after this engagement based on
      the PKUs,"
3000 LPRINT "          EXPAJ is an estimate of the expected
      value for NEWAJ calculated as DENAJ*NEWAJ,"
3010 LPRINT "          SUMEK is the sum EXPAJ + TOTKL,"
3020 LPRINT "          PROP KILL is the proportional kill
      calculated as TOTKL/SUMEK,"
3030 LPRINT "          METH is the analysis method,"
3040 LPRINT "          REAL KILL is the simulated RTCA result"
3050 LPRINT "          (KILL<0 means no assessment, 0<=KILL<1
      means assessment but no kill, and KILL>=1 means RTCA kill)"
3060 LPRINT "          (values of KILL with a
      trailing .5 indicate engagements where Method 1 gave a negative
      A(J))."
3070 LPRINT "          ENGJ is the cumulative number of
      engagements in which player J was the Firer (F) or Target (T),"
3080 LPRINT "          BLUE and RED are the numbers of
      Blue and Red SURVIVORS remaining alive in RTCA."
3090 LPRINT: LPRINT "          The Engagement Summaries are self
      explanatory except that:
3100 LPRINT "          PLAYER STS is the current real time
      dead/alive status of the player (0=dead, 1=alive),"
3110 LPRINT "          ALIVENESS WGHT is the current value
      of A(J) while ALIVENESS EXPT is the current value of EXPAJ,"
3120 LPRINT "          OVERALL PROPORTION KILLED is computed
      as (TOTAL TIMES KILLD)/(TOTAL ALIVENESS EXPT + TOTAL TIMES KILLD)"
3130 LPRINT "          (notice that the proportion
      of the blue/red force credited as killed appears as the MEAN
      TIMES KILLD)."
3140 RETURN
3150 REM***
3160 REM*** SUBROUTINE WHICH PRINTS ITERATION SUMMARY
3170 REM***
3180 GOSUB 2130: PLS="BLUE"
3190 MST=1: MEND=BLUE: PLAYNUM=BLUE: GOSUB 3230
3200 PRINT #1, : PRINT: PLS="RED"
3210 MST=BLUE+1: MEND=TOTAL: PLAYNUM=RED: GOSUB 3230
3220 RETURN
3230 REM***
3240 REM*** SUBROUTINE WHICH SUPPORTS PRINTING OF ITERATION SUMMARY
3250 REM***
3260 PRINT: PRINT USING "          NUMBR"; PLS; : FOR K=1 TO MTH:
      PRINT USING "    === METHOD # ==="; K; : NEXT K
3270 PRINT #1, : PRINT #1, USING "          NUMBR  AVG PROB KILL ";
      PLS; : FOR K=1 TO MTH: PRINT #1, USING "    ===== METHOD #
      ====="; K; : NEXT K
3280 PRINT: PRINT "PLAYER  ENGMT"; : FOR K=1 TO MTH:
      PRINT "    TOTAL TIMES ALIVE"; : NEXT K
3290 PRINT #1, : PRINT #1, "PLAYER  ENGMT  FIRER  TARGET "; : FOR K=1
      TO MTH: PRINT #1, "    TOTAL TIMES  ALIVENESS  PROP"; : NEXT K
3300 PRINT: PRINT "ID STS  F  T"; : FOR K=1 TO MTH:
      PRINT "    KILLS KILLD -NESS"; : NEXT K: PRINT
3310 PRINT #1, : PRINT #1, "ID STS  F  T PKA PKU PKA PKU"; :
      FOR K=1 TO MTH: PRINT #1, "    KILLS KILLD  WGHT  EXPT  KILL"; :
      NEXT K: PRINT #1,
3320 FOR K=1 TO MTH: KDCNT(K)=0: KSCNT(K)=0: A(0,K)=0: ALTOT(K)=0:
      EXPA(0,K)=0: RAT(K)=0: NEXT K: FOR FT=0 TO 1: ENG(0,FT)=0:
      SPKA(0,FT)=0: SPKU(0,FT)=0: NE
3330 FOR M=MST TO MEND: FOR FT=0 TO 1: ENG(0,FT)=ENG(0,FT)+ENG(M,FT):
      SPKA(0,FT)=SPKA(0,FT)+SPKA(M,FT):

```

```

3340 IF ENG(M,FT)=0 THEN APKA(FT)=0: APKU(FT)=0: ELSE APKA(FT)=
      SPKA(M,FT)/ENG(M,FT): APKU(FT)=SPKU(M,FT)/ENG(M,FT)
3350 NEXT FT
3360 FOR K=1 TO 3: KDCNT(K)=KDCNT(K)+K(O,M,K): KSCNT(K)=KSCNT(K)+K(M,O,K):
      A(O,K)=A(O,K)+A(M,K): ALTOT(K)=ALTOT(K)+STS(M)*A(M,K):
      EXPA(O,K)=EXPA(O,K)+EXPA(M,K): RAT(K)=RAT(K)+RATIO(M,K): NEXT K
3370 PRINT: PRINT USING "## # ## ##"; M; STS(M); ENG(M,O); ENG(M,1);:
      FOR K=1 TO MTH: PRINT USING "### ## ##.###";
      K(M,O,K); K(O,M,K); A(M,K);: NEXT K
3380 PRINT #1,: PRINT #1, USING "## # ## ## ##.###.###.###"; M;
      STS(M); ENG(M,O); ENG(M,1); APKA(O); APKU(O); APKA(1); APKU(1);:
      FOR K=1 TO MTH: LPRINT USING "### ## ##.###.###.###.###";
      K(M,O,K); K(O,M,K); A(M,K); EXPA(M,K); RATIO(M,K)
3390 NEXT K, M
3400 PRINT: PRINT USING "TOTAL ### ##"; ENG(O,O); ENG(O,1);:
      FOR K=1 TO MTH: PRINT USING "### ## ##.###"; KDCNT(K);
      KSCNT(K); A(O,K);: NEXT K: PRINT
3410 PRINT #1,: PRINT #1, USING "TOTAL ### ## " ;
      ENG(O,O); ENG(O,1);: FOR K=1 TO MTH: PRINT #1, USING "### ##
      ##.### ##.### ##.###"; KSCNT(K); KDCNT(K); A(O,K);
      EXPA(O,K); RAT(K);: NEXT K
3420 FOR FT=0 TO 1: IF ENG(O,FT)=0 THEN APKA(FT)=0: APKU(FT)=0:
      ELSE APKA(FT)=SPKA(O,FT)/ENG(O,FT): APKU(FT)=SPKU(O,FT)/ENG(O,FT)
3430 NEXT FT
3440 PRINT #1,: PRINT #1, USING "MEAN ##.##.##.##.##.###";
      ENG(O,O)/PLAYNUM; ENG(O,1)/PLAYNUM; APKA(O); APKU(O); APKA(1);
      APKU(1);
3450 FOR K=1 TO MTH: PRINT #1, USING "### ## ##.###.###.###.###
      ##.###"; KSCNT(K)/PLAYNUM; KDCNT(K)/PLAYNUM; A(O,K)/PLAYNUM;
      EXPA(O,K)/PLAYNUM; RAT(K)/PLAYNUM;: NEXT K
3460 PRINT #1,: PRINT #1, "OVERALL PROPORTION KILLED " ;:
      FOR K=1 TO MTH: PRINT #1, USING "###.###";
      KDCNT(K)/(EXPA(O,K)+KDCNT(K));: NEXT K
3470 PRINT #1,: PRINT #1, "TIMES KILLED + ALIVENESS EXPT " ;:
      FOR K=1 TO MTH: PRINT #1, USING "###.### " ;
      EXPA(O,K)+KDCNT(K);: NEXT K: LPRINT
3480 REM *** PRINT "Total of Live";: FOR K=1 TO MTH: PRINT USING
      "###.###"; ALTOT(K);: NEXT K: PRINT
3490 REM *** LPRINT "TOTAL ALIVENESS OF LIVE PLAYERS";:
      FOR K=1 TO MTH: LPRINT USING "###.### " ;
      ALTOT(K);: NEXT K: LPRINT
3500 RETURN

```

## APPENDIX B

### FIRST EXPERIMENT HISTOGRAMS

This appendix contains the four frequency histograms for the first experiment. Each histogram represents one hundred forty iterations.

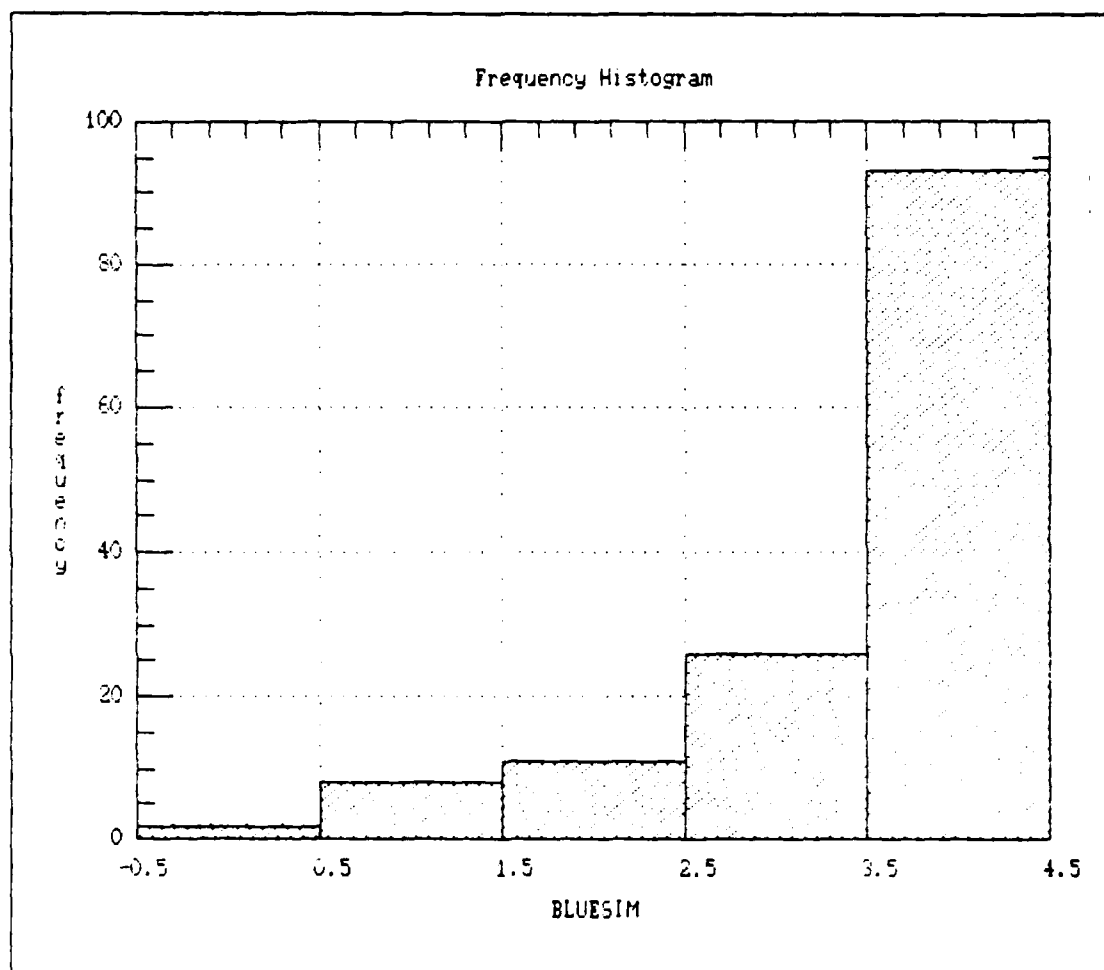


Figure B.1 Blue Casualties by Simulation.

Figure B.1 displays the frequency histogram for the number of blue casualties for one hundred forty iterations of simulations with the PKUs as the standard pair ( $BPKU = 0.5$ ,  $RPKU = 0.2$ ). The summary statistics are listed in Table 6.

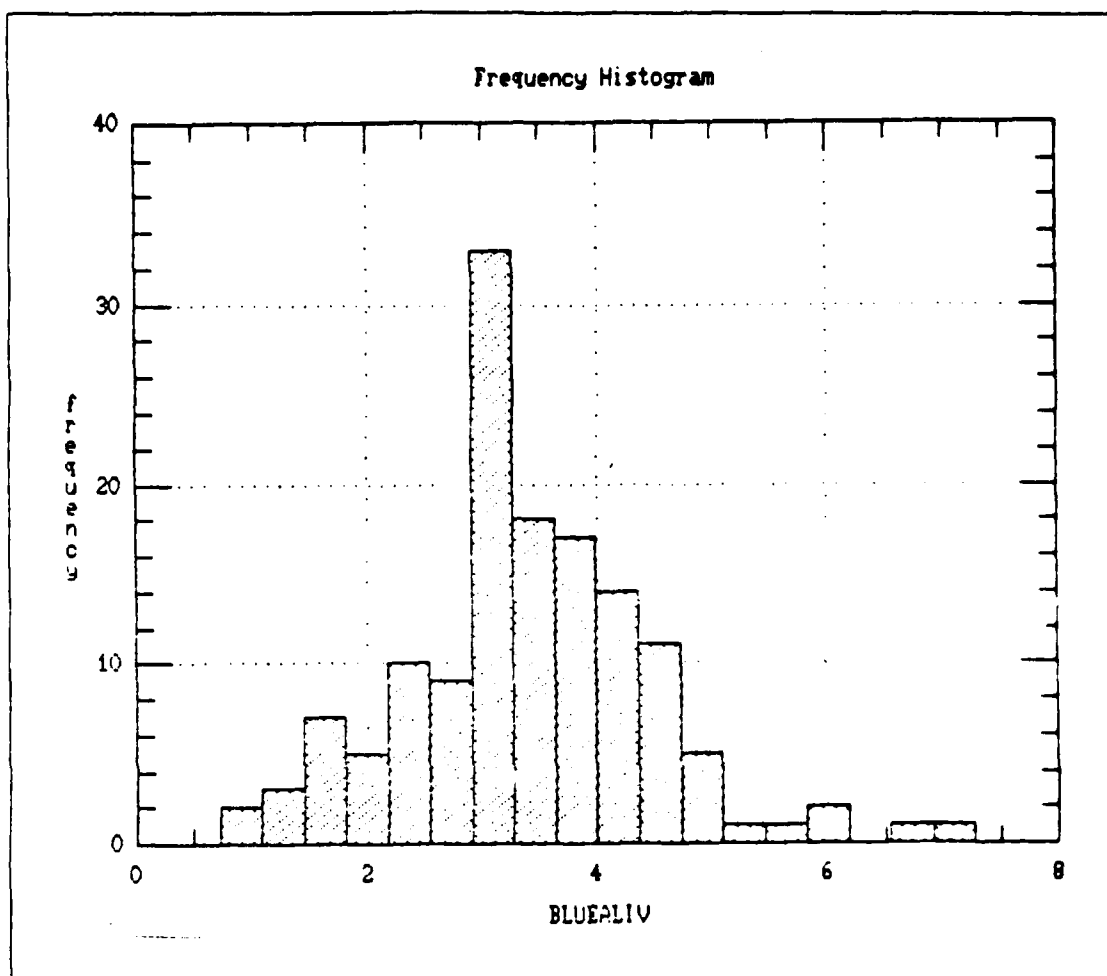


Figure B.2 Blue Casualties by Aliveness.

Figure B.2 displays the frequency histogram for the number of blue casualties for one hundred forty iterations of simulations with the PKAs as the standard pair ( $BPKA = 0.5$ ,  $RPKA = 0.2$ ). The summary statistics are listed in Table 6.

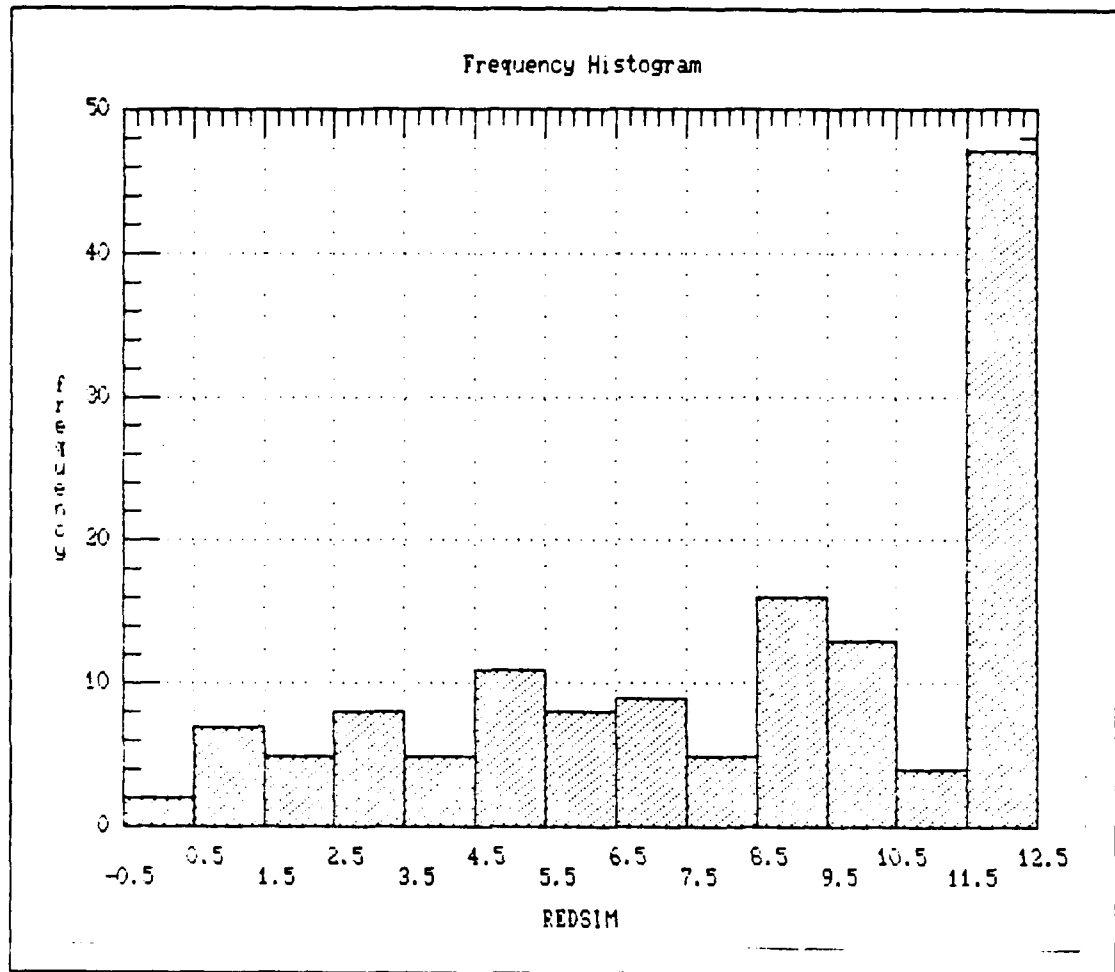


Figure B.3 Red Casualties by Simulation.

Figure B.3 displays the frequency histogram for the number of red casualties for one hundred forty iterations of simulations with the PKUs as the standard pair ( $BPKU = 0.5$ ,  $RPKU = 0.2$ ). The summary statistics are listed in Table 6.

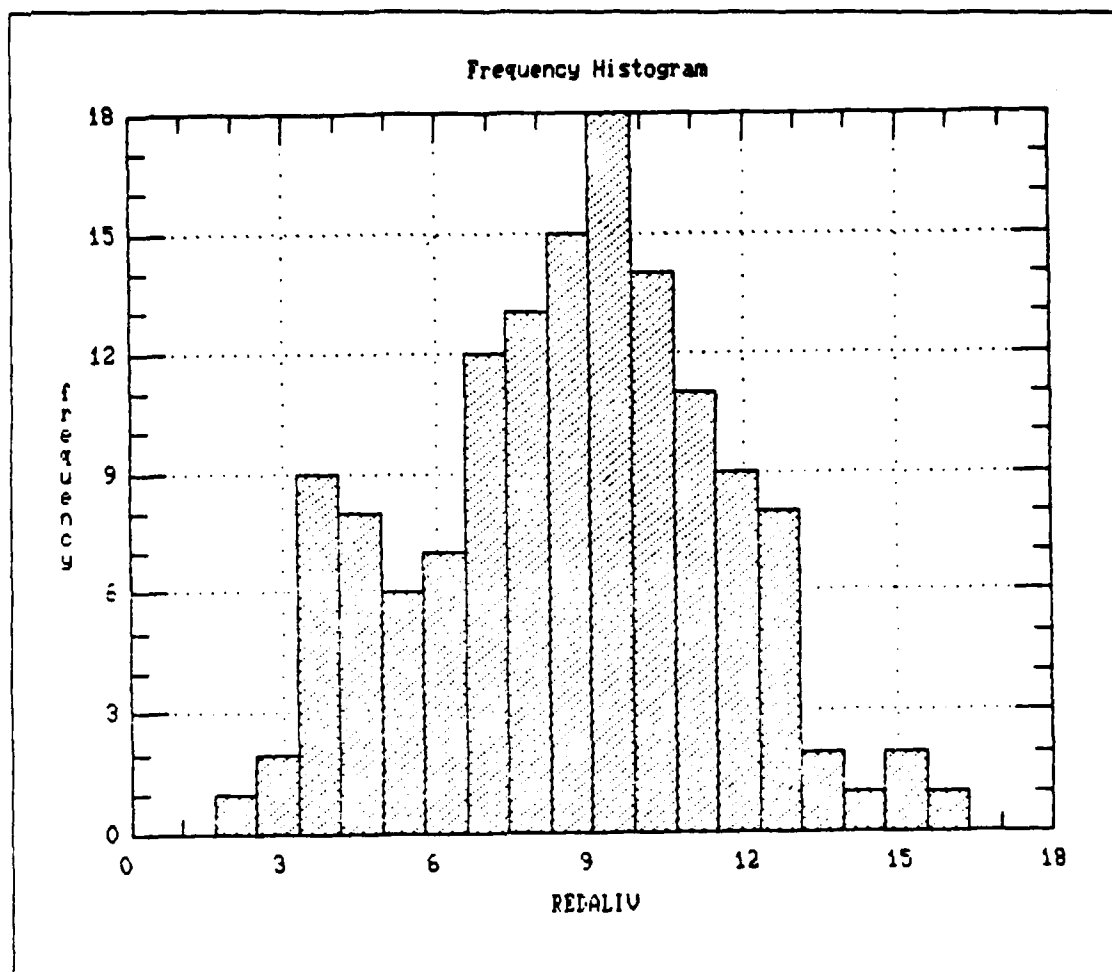


Figure B.4 Red Casualties by Aliveness.

Figure B.4 displays the frequency histogram for the number of red casualties for one hundred forty iterations of simulations with the PKAs as the standard pair ( $BPKA = 0.5$ ,  $RPKA = 0.2$ ). The summary statistics are listed in Table 6.

## APPENDIX C

### TARGET SELECTION METHOD BY ALIVENESS

This appendix contains the modification to the target selection method to allow the firer to select the player on the other side with the greatest aliveness value (TGTSSELMETH 2).

```

1400 REM***
1410 REM*** CHOOSE FIRER AND TARGET
1420 REM***
1430 IF RND<.5 THEN I1=INT(RND*BNUM+1) ELSE I1=INT(RND*RNUM+RPOS)
1440 I=PLAYER(I1)
1445 TEMPTGT=-1
1446 IF I1>BNUM THEN GOTO 1454
1447 FOR TGT=RPOS TO TOT
1448 AB=PLAYER(TGT)
1449 IF A(AB,3)<TEMPTGT THEN GOTO 1452
1450 REM***IF I1>BNUM THEN J1=INT(RND*BNUM+1) ELSE J1=INT(RND*RNUM+RPOS)
1451 TEMPTGT=A(AB,3): J=AB: AZ=TGT
1452 NEXT TGT
1453 GOTO 1470
1454 FOR TGT=1 TO BNUM
1455 AB=PLAYER(TGT)
1456 IF A(AB,3) < TEMPTGT THEN GOTO 1458
1457 TEMPTGT=A(AB,3): J=AB: AZ=TGT
1458 NEXT TGT
1459 GOTO 1470
1460 REM*** J=PLAYER(J1)
1461 REM*** do not forget to change lines 1600 and 1610 variable J1 to AB
1470 ENG(I,0)=ENG(I,0)+1: ENG(J,1)=ENG(J,1)+1
1480 REM***
1490 REM*** FIND PKs AND ASSESS REAL TIME KILLS
1500 REM***
1510 RTKILL=0
1520 PKA=PK(I,J,1)
1530 PKU=PK(I,J,2)
1540 IF POPT<2 GOTO 1570
1550 IF J>BNUM THEN JIT=BJIT ELSE JIT=RJIT
1560 PJIT=PKA: GOSUB 2690: PKA=PJIT: PJIT=PKU: GOSUB 2690: PKU=PJIT
1570 IF RND<PKGOOF THEN PKU=0: RTKILL=-1
1580 SPKA(I,0)=SPKA(I,0)+PKA: SPKA(J,1)=SPKA(J,1)+PKA
1590 SPKU(I,0)=SPKU(I,0)+PKU: SPKU(J,1)=SPKU(J,1)+PKU
1600 IF RND>PKU GOTO 1650 ELSE RTKILL=1: STS(J)=0:
      IF AZ>BNUM THEN RNUM=RNUM-1 ELSE RPOS=RPOS-1: BNUM=BNUM-1
1610 IF AZ<TOT THEN FOR L=AZ TO TOT-1: PLAYER(L)=PLAYER(L+1): NEXT L
1620 REM***

```

## APPENDIX D

### TARGET SELECTION METHOD BY FIRER'S PKU

This appendix contains the modification to the target selection method to allow the firer to select the player on the other side against which he has the greatest used probability of kill, PKU (TGTSELMETH 3).

```

1400 REM***
1410 REM*** CHOOSE FIRER AND TARGET
1420 REM***
1430 IF RND<.5 THEN I1=INT(RND*BNUM+1) ELSE I1=INT(RND*RNUM+RPOS)
1440 I=PLAYER(I1)
1445 TEMPTPK=0!
1446 IF I1>BNUM THEN GOTO 1454
1447 FOR TGT=RPOS TO TOT
1448 AB=PLAYER(TGT)
1449 IF PK(I,AB,2)<TEMPTPK THEN GOTO 1452
1450 REM***IF I1>BNUM THEN J1=INT(RND*BNUM+1) ELSE J1=INT(RND*RNUM+RPOS)
1451 TEMPTPK=PK(I,AB,2): J=AB: AZ=TGT
1452 NEXT TGT
1453 GOTO 1470
1454 FOR TGT=1 TO BNUM
1455 AB=PLAYER(TGT)
1456 IF PK(I,AB,2)<TEMPTPK THEN GOTO 1458
1457 TEMPTPK=PK(I,AB,2): J=AB: AZ=TGT
1458 NEXT TGT
1459 GOTO 1470
1460 REM*** J=PLAYER(J1)
1461 REM*** do not forget to change lines 1600 and 1610 variable J1 to AB
1470 ENG(I,0)=ENG(I,0)+1: ENG(J,1)=ENG(J,1)+1
1480 REM***
1490 REM*** FIND PKs AND ASSESS REAL TIME KILLS
1500 REM***
1510 RTKILL=0
1520 PKA=PK(I,J,1)
1530 PKU=PK(I,J,2)
1540 IF POPT<2 GOTO 1570
1550 IF J>BNUM THEN JIT=BJIT ELSE JIT=RJIT
1560 PJIT=PKA: GOSUB 2690: PKA=PJIT: PJIT=PKU: GOSUB 2690: PKU=PJIT
1570 IF RND<PKGOOF THEN PKU=0: RTKILL=-1
1580 SPKA(I,0)=SPKA(I,0)+PKA: SPKA(J,1)=SPKA(J,1)+PKA
1590 SPKU(I,0)=SPKU(I,0)+PKU: SPKU(J,1)=SPKU(J,1)+PKU
1600 IF RND>PKU GOTO 1650 ELSE RTKILL=1: STS(J)=0:
      IF AZ>BNUM THEN RNUM=RNUM-1 ELSE RPOS=RPOS-1: BNUM=BNUM-1
1610 IF AZ<TOT THEN FOR L=AZ TO TOT-1: PLAYER(L)=PLAYER(L+1): NEXT L
1620 REM***

```

## APPENDIX E

### TARGET SELECTION METHOD BY TARGET'S PKU

This appendix contains the modification to the target selection method to allow the firer to select the player on the other side which has the greatest used probability of kill (PKU) to kill the firer (TGTSELMETH 4).

```
1400 REM***
1410 REM*** CHOOSE FIRER AND TARGET
1420 REM***
1430 IF RND<.5 THEN I1=INT(RND*BNUM+1) ELSE I1=INT(RND*RNUM+RPOS)
1440 I=PLAYER(I1)
1445 TEMPTPK=0!
1446 IF I1>BNUM THEN GOTO 1454
1447 FOR TGT=RPOS TO TOT
1448 AB=PLAYER(TGT)
1449 IF PK(AB,I,2)<TEMPTPK THEN GOTO 1452
1450 REM***IF I1>BNUM THEN J1=INT(RND*BNUM+1) ELSE J1=INT(RND*RNUM+RPOS)
1451 TEMPTPK=PK(AB,I,2): J=AB: AZ=TGT
1452 NEXT TGT
1453 GOTO 1470
1454 FOR TGT=1 TO BNUM
1455 AB=PLAYER(TGT)
1456 IF PK(AB,I,2)<TEMPTPK THEN GOTO 1458
1457 TEMPTPK=PK(AB,I,2): J=AB: AZ=TGT
1458 NEXT TGT
1459 GOTO 1470
1460 REM*** J=PLAYER(J1)
1461 REM*** do not forget to change lines 1600 and 1610 variable J1 to AB
1470 ENG(I,0)=ENG(I,0)+1: ENG(J,1)=ENG(J,1)+1
1480 REM***
1490 REM*** FIND PKs AND ASSESS REAL TIME KILLS
1500 REM***
1510 RTKILL=0
1520 PKA=PK(I,J,1)
1530 PKU=PK(I,J,2)
1540 IF POPT<2 GOTO 1570
1550 IF J>BNUM THEN J1=BJIT ELSE J1=RJIT
1560 PJIT=PKA: GOSUB 2690: PKA=PJIT: PJIT=PKU: GOSUB 2690: PKU=PJIT
1570 IF RND<PKG00F THEN PKU=0: RTKILL=-1
1580 SPKA(I,0)=SPKA(I,0)+PKA: SPKA(J,1)=SPKA(J,1)+PKA
1590 SPKU(I,0)=SPKU(I,0)+PKU: SPKU(J,1)=SPKU(J,1)+PKU
1600 IF RND>PKU GOTO 1650 ELSE RTKILL=1: STS(J)=0:
    IF AZ>BNUM THEN RNUM=RNUM-1 ELSE RPOS=RPOS-1: BNUM=BNUM-1
1610 IF AZ<TOT THEN FOR L=AZ TO TOT-1: PLAYER(L)=PLAYER(L+1): NEXT L
1620 REM***
```

## APPENDIX F

### TWO-FACTOR INTERACTIONS FOR RED CASUALTIES

This appendix contains the two multiple x-y plots for the two-factor interactions for mean red casualties by both the simulation and aliveness methods. The extraneous (and misleading) probability of kill pairs (PKPAIRs) have been removed to evaluate the interactions with only equitable PKPAIRs.

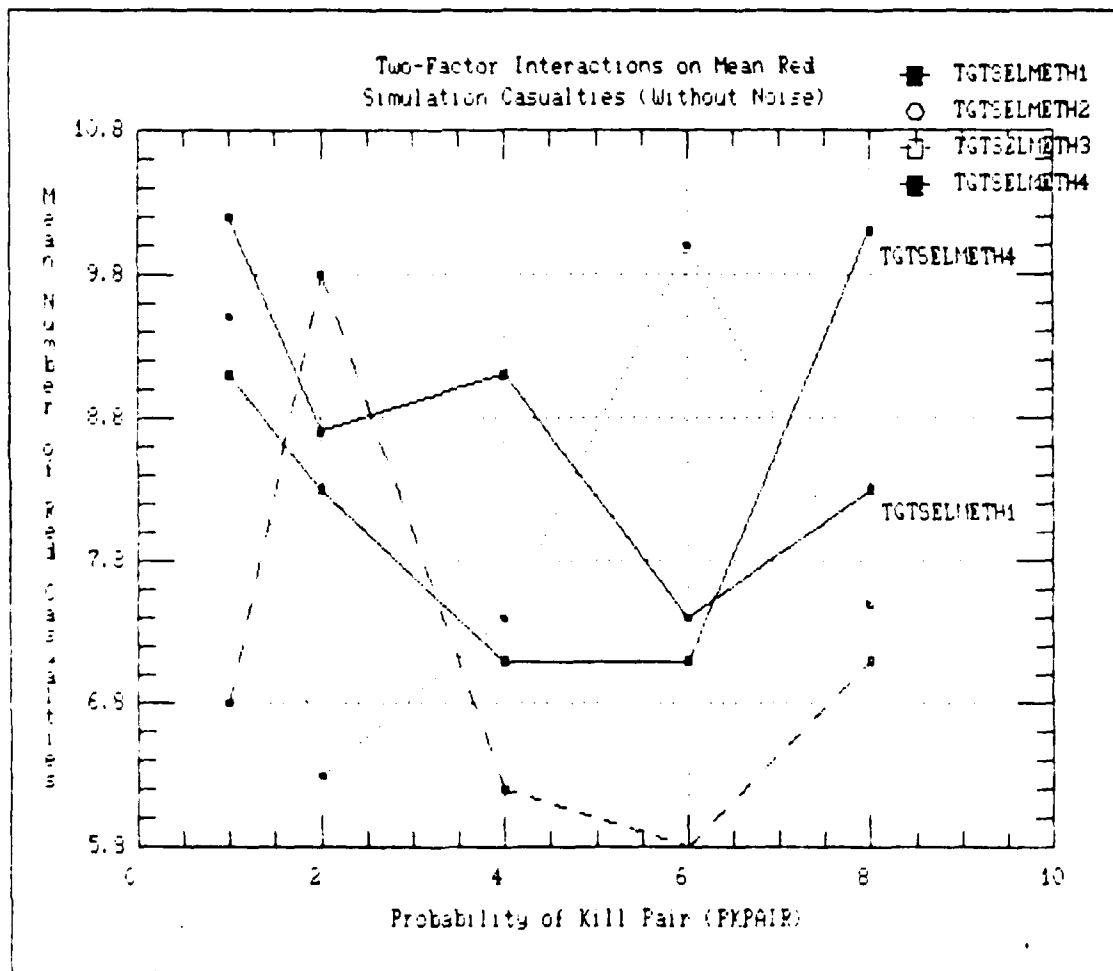


Figure F.1 Two-Factor Interactions of Mean Red Simulation Casualties.

Figure F.1 displays the mean number of simulated red casualties as a function of applicable PKPAIRs for the four different target selection methods (TGTSELMETH).

Each of the thirty-six points on the plot represent the mean number of red casualties for ten iterations. All of the simulation results plotted in Figure F.1 used the standard probability of kill ( $BPKU = 0.5$  and  $RPKU = 0.2$ ).

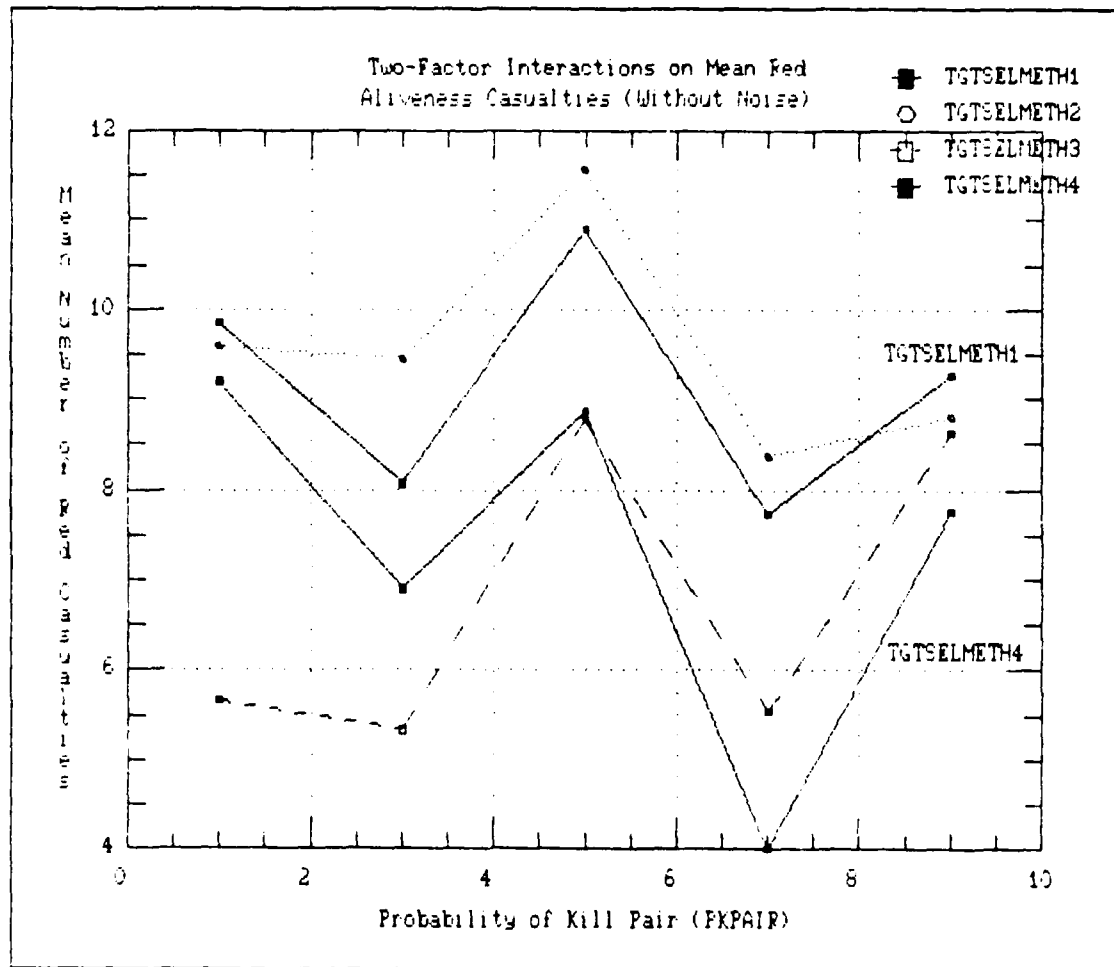


Figure F.2 Two-Factor Interactions of Mean Red Aliveness Casualties.

Figure F.2 displays the mean number of red casualties using the aliveness method as a function of applicable PKPAIRs for the four different target selection methods (TGTSELMETH). Each of the thirty-six points on the plot represent the mean number of red casualties for ten iterations. All of the aliveness method results plotted in Figure F.2 adjusted the simulation data to the standard probability of kill ( $BPKA = 0.5$  and  $RPKA = 0.2$ ).

## APPENDIX G

### SUPPORTING DIAGRAMS FOR THE DIFFERENCES IN CASUALTIES

This appendix contains the notched box plots and two-factor interaction plots for differences in casualties analysis conducted as part of the second experiment.

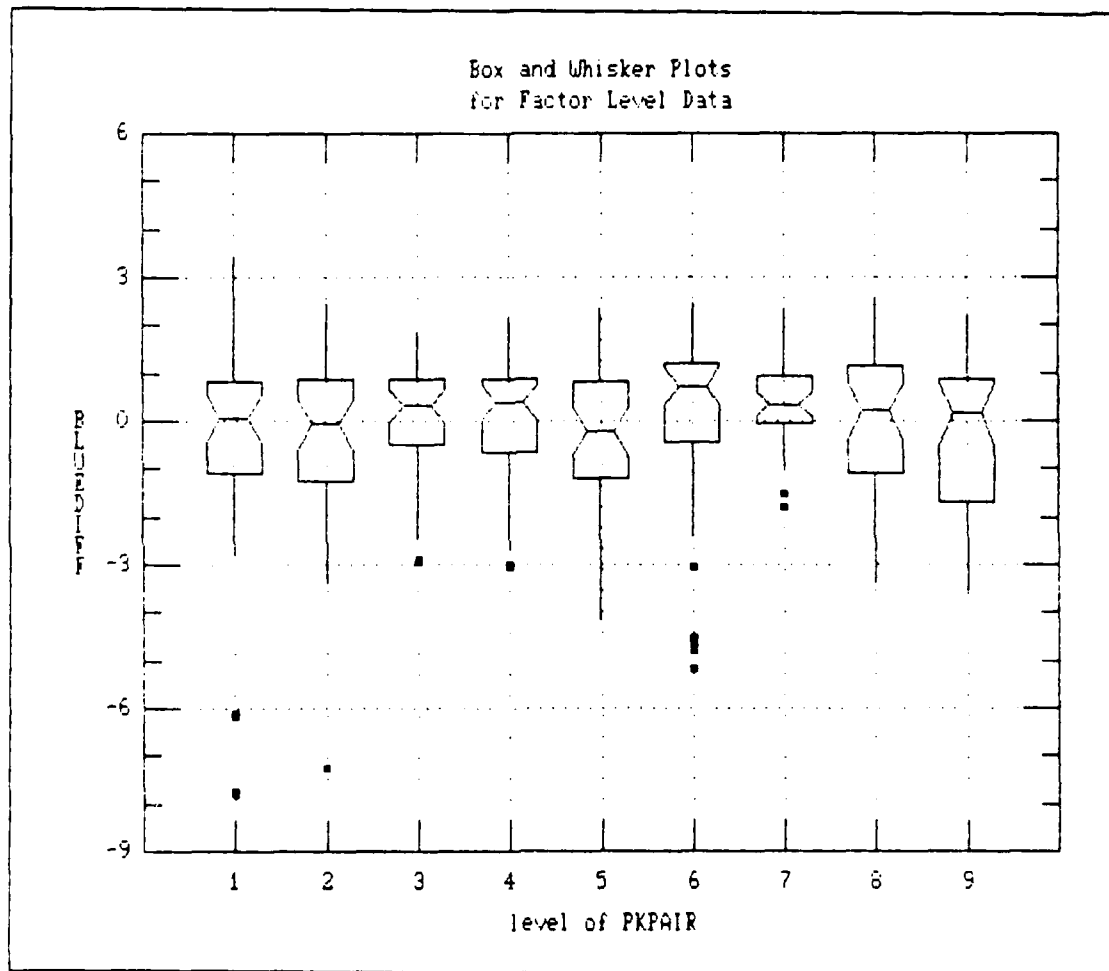


Figure G.1 Notched Box Plots of BLUEDIFF as a Function of PKPAIR.

Figure G.1 displays the notched box plots of the differences in blue casualties (BLUEDIFF) as a function of probability of kill pairs (PKPAIRs). BLUEDIFF is the estimated number of expected blue casualties using the aliveness method subtracted from the estimated number of expected blue casualties using the corresponding

simulation. There are forty differences (ten differences for each of the four target selection methods) represented by each notched box. The "waist" indicates the mean and the length of the box is the middle fifty percent.

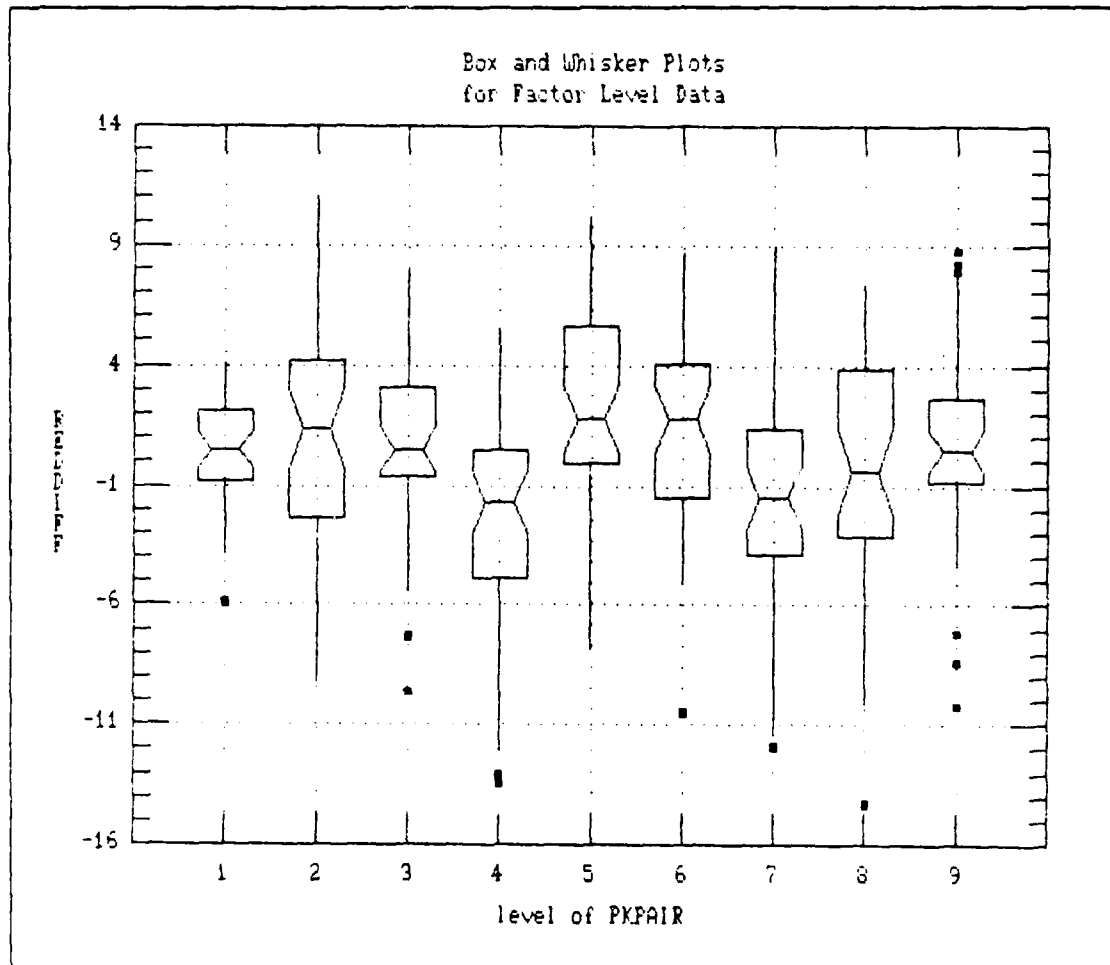


Figure G.2 Notched Box Plots of REDDIFF as a Function of PKPAIR.

Figure G.2 displays the notched box plots of the differences in red casualties (REDDIFF) as a function of probability of kill pairs (PKPAIRs). REDDIFF is the estimated number of expected red casualties using the aliveness method subtracted from the estimated number of expected red casualties using the corresponding simulation. There are forty differences (ten differences for each of the four target selection methods) represented by each notched box. The "waist" indicates the mean and the length of the box is the middle fifty percent.

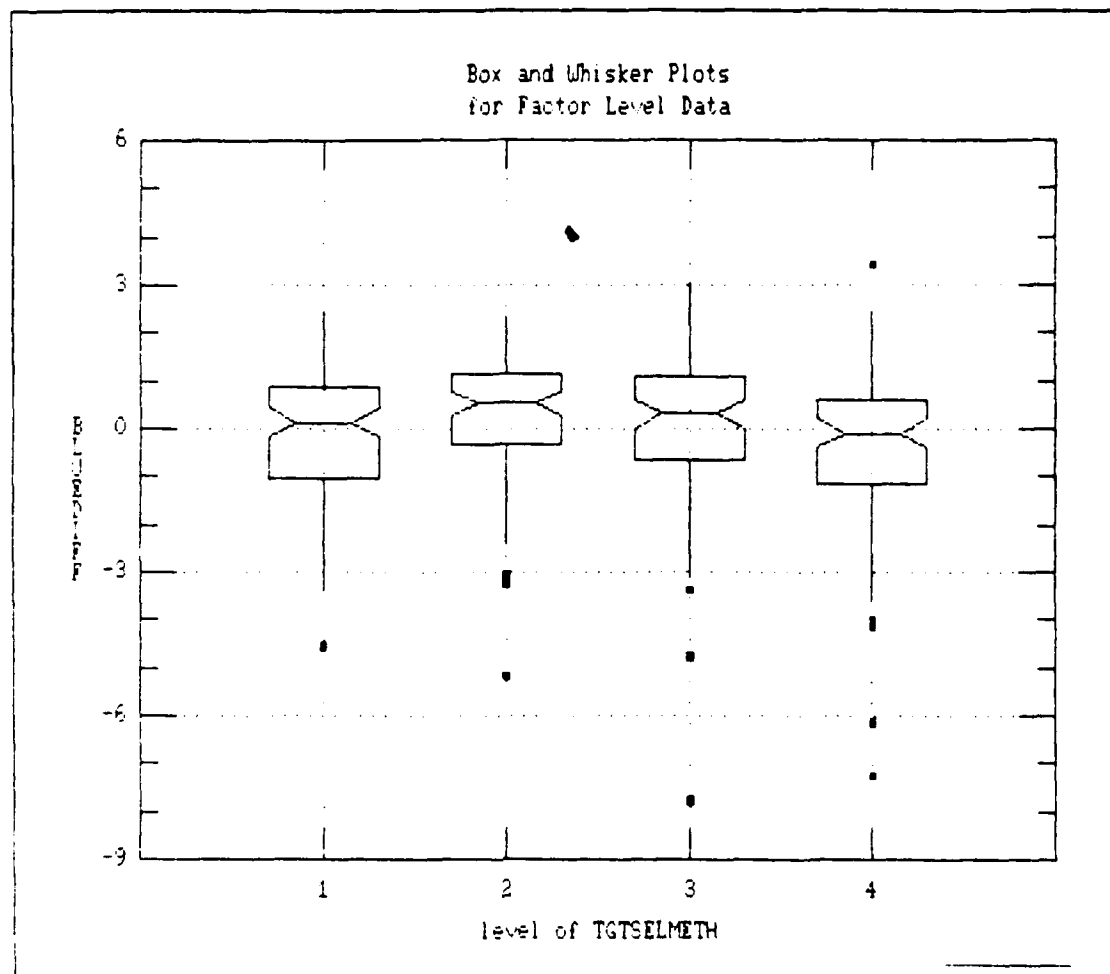


Figure G.3 Notched Box Plots of BLUEDIFF as a Function of TGTSELMETH.

Figure G.3 displays the notched box plots of the differences in blue casualties (BLUEDIFF) as a function of target selection method (TGTSELMETH). BLUEDIFF is the estimated number of expected blue casualties using the aliveness method subtracted from the estimated number of expected blue casualties using the corresponding simulation. There are ninety differences (ten differences for each of the nine probability of kill pairs) represented by each notched box. The "waist" indicates the mean and the length of the box is the middle fifty percent.

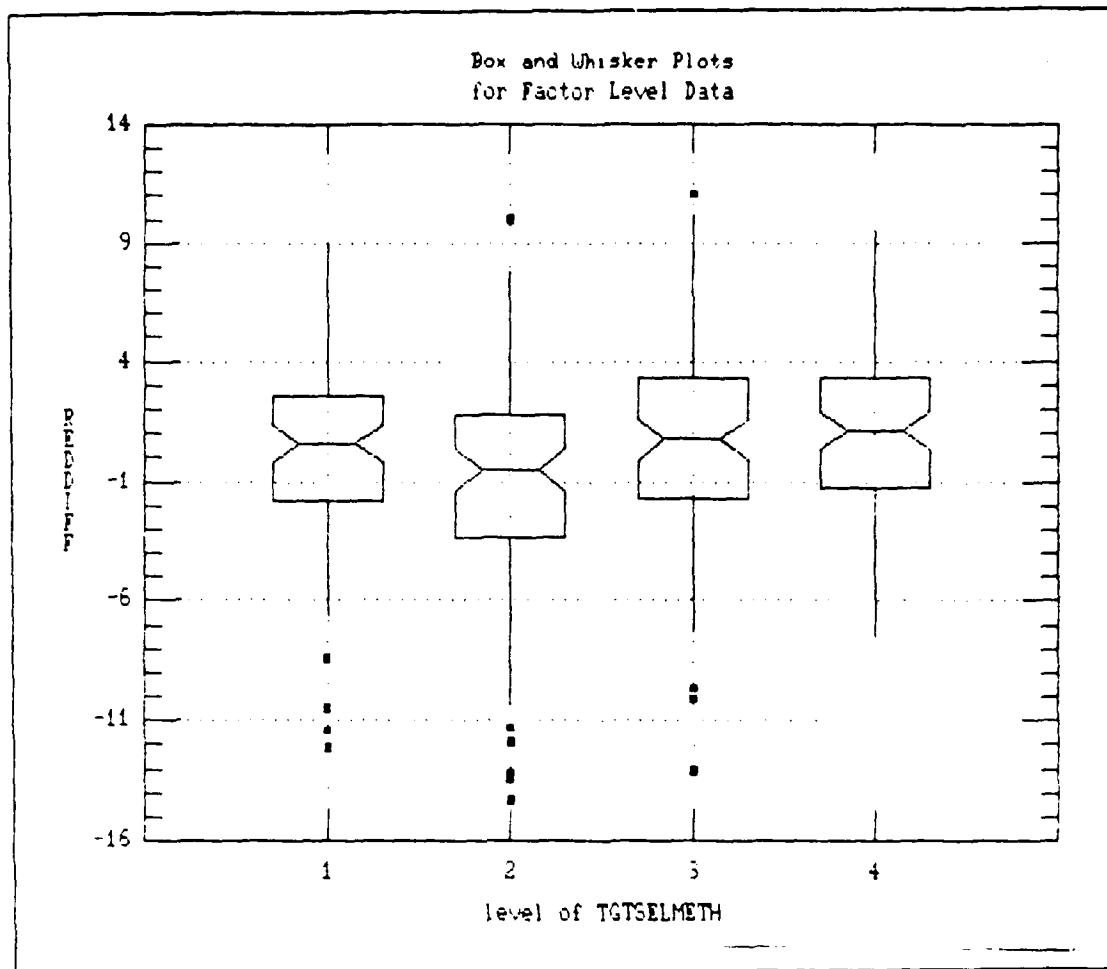


Figure G.4 Notched Box Plots of REDDIFF as a Function of TGTSELMETH.

Figure G.4 displays the notched box plots of the differences in red casualties (REDDIFF) as a function of target selection method (TGTSELMETH). REDDIFF is the estimated number of expected red casualties using the aliveness method subtracted from the estimated number of expected red casualties using the corresponding simulation. There are ninety differences (ten differences for each of the nine probability of kill pairs) represented by each notched box. The "waist" indicates the mean and the length of the box is the middle fifty percent.

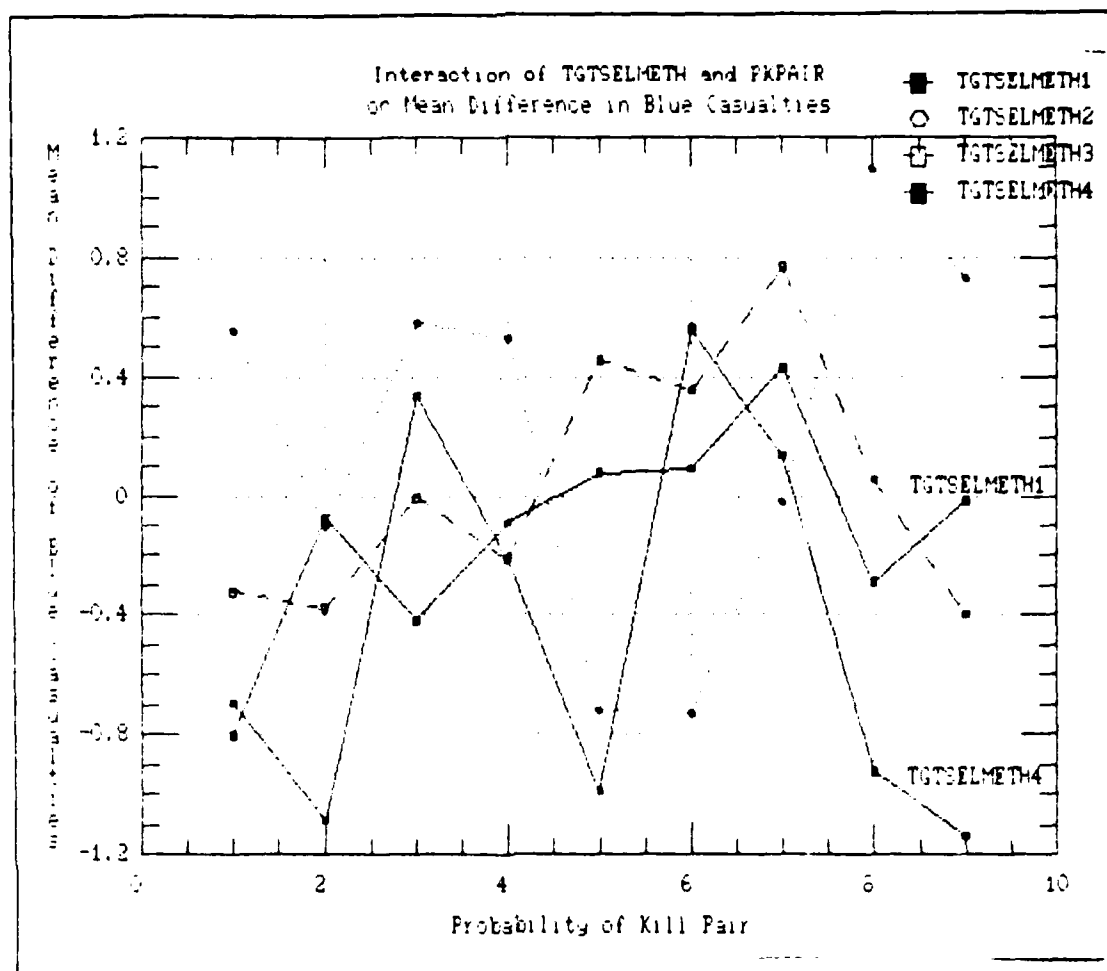


Figure G.5 Two-Factor Interactions of BLUEDIFF.

Figure G.5 displays the mean difference in blue casualties (BLUEDIFF) as a function of probability of kill pairs (PKPAIRs) for the different target selection methods (TGTSELMETH). BLUEDIFF is the estimated number of expected blue casualties using the aliveness method subtracted from the estimated number of expected blue casualties using the corresponding simulation. Each of the thirty six points on the plot represent the mean of ten differences of blue casualties.

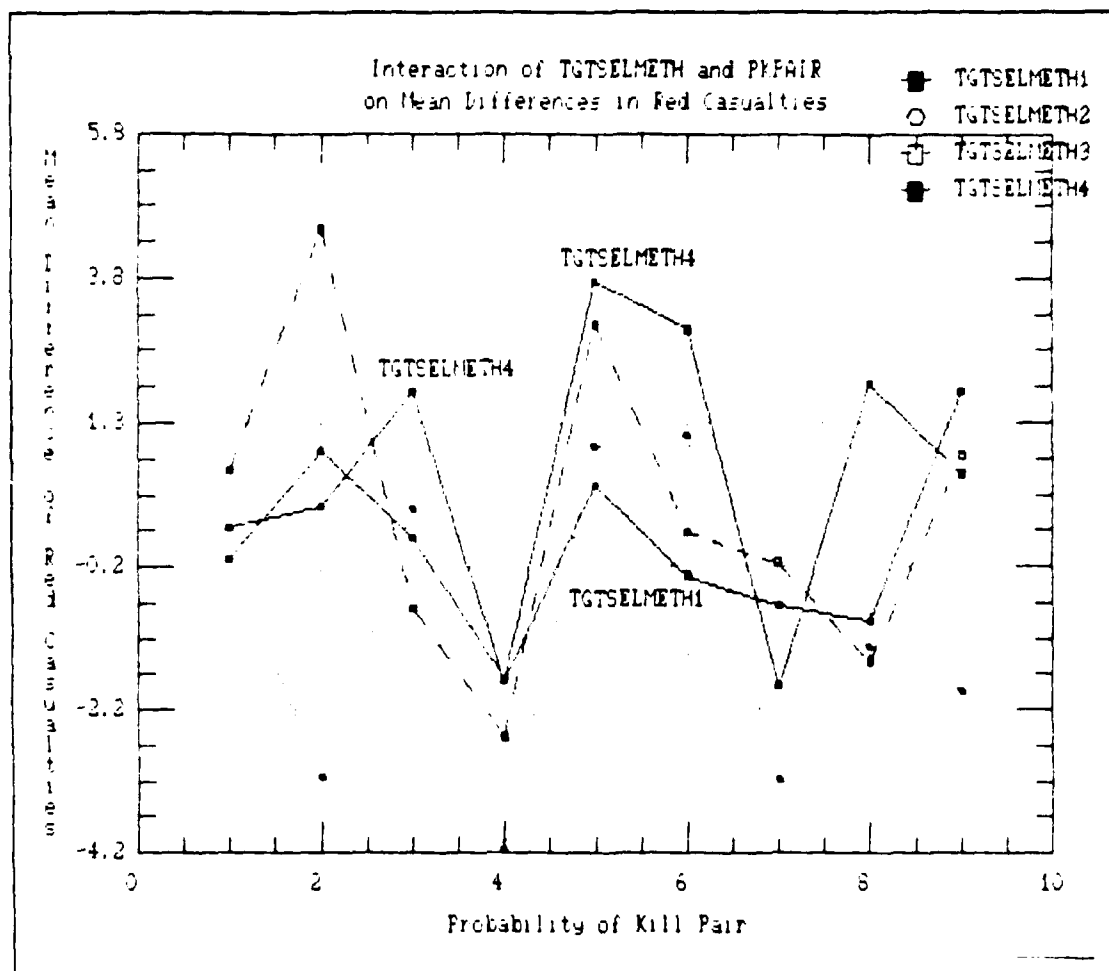


Figure G.6 Two-Factor Interactions of REDDIFF.

Figure G.6 displays the mean difference in red casualties (REDDIFF) as a function of probability of kill pairs (PKPAIRs) for the different target selection methods (TGTSELMETH). REDDIFF is the estimated number of expected red casualties using the aliveness method subtracted from the estimated number of expected red casualties using the corresponding simulation. Each of the thirty six points on the plot represent the mean of ten differences of red casualties.

## APPENDIX H

### SUPPORTING FIGURES FOR THE SQRTSSD

This appendix contains the supporting figures for the residual plot and notched box plots for the square root of the squared differences in estimated casualties (SQRTSSD) conducted as part of the second experiment.

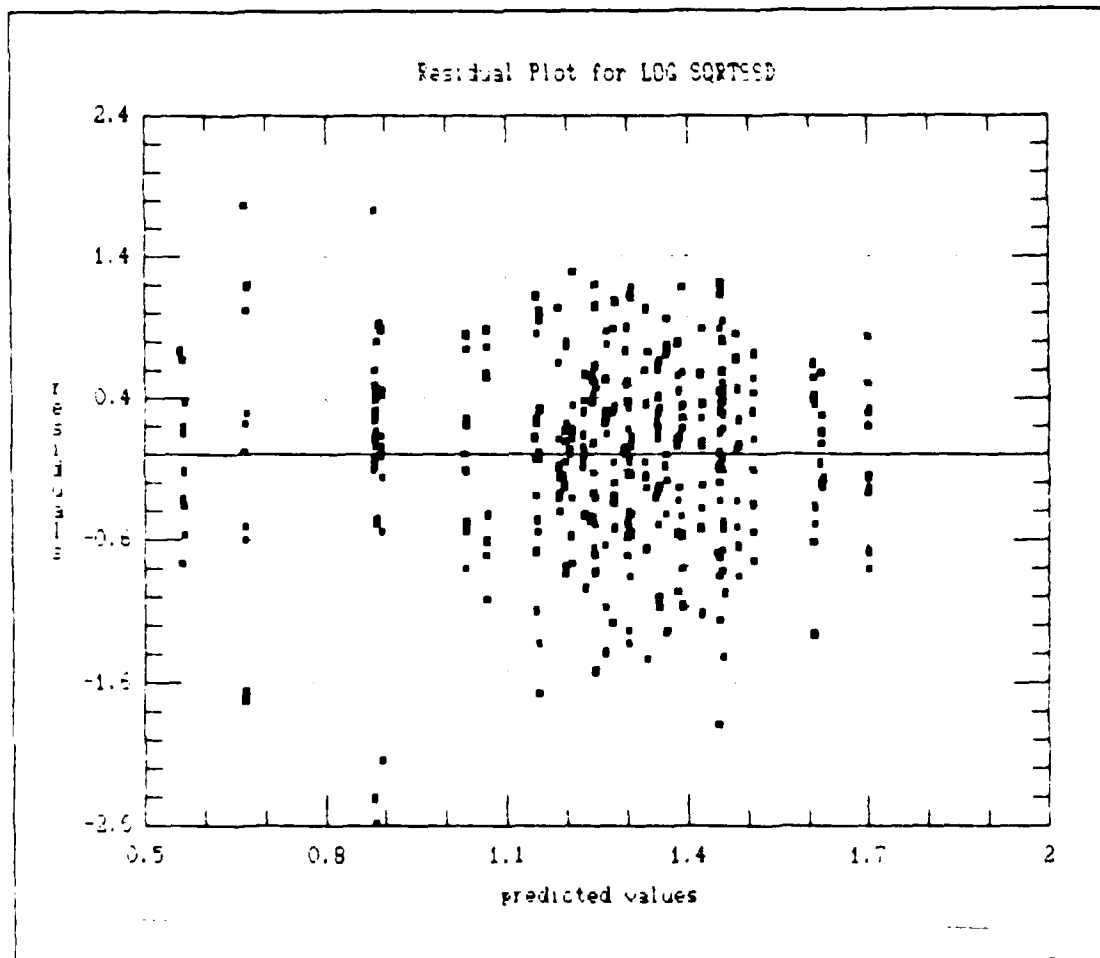


Figure H.1 Plot of LOG SQRTSSD Residuals Against Predicted Values.

Figure H.1 displays the residuals of the logarithmic transformation of the square root of the sum of the squared differences (LOG SQRTSSD) against the predicted values.

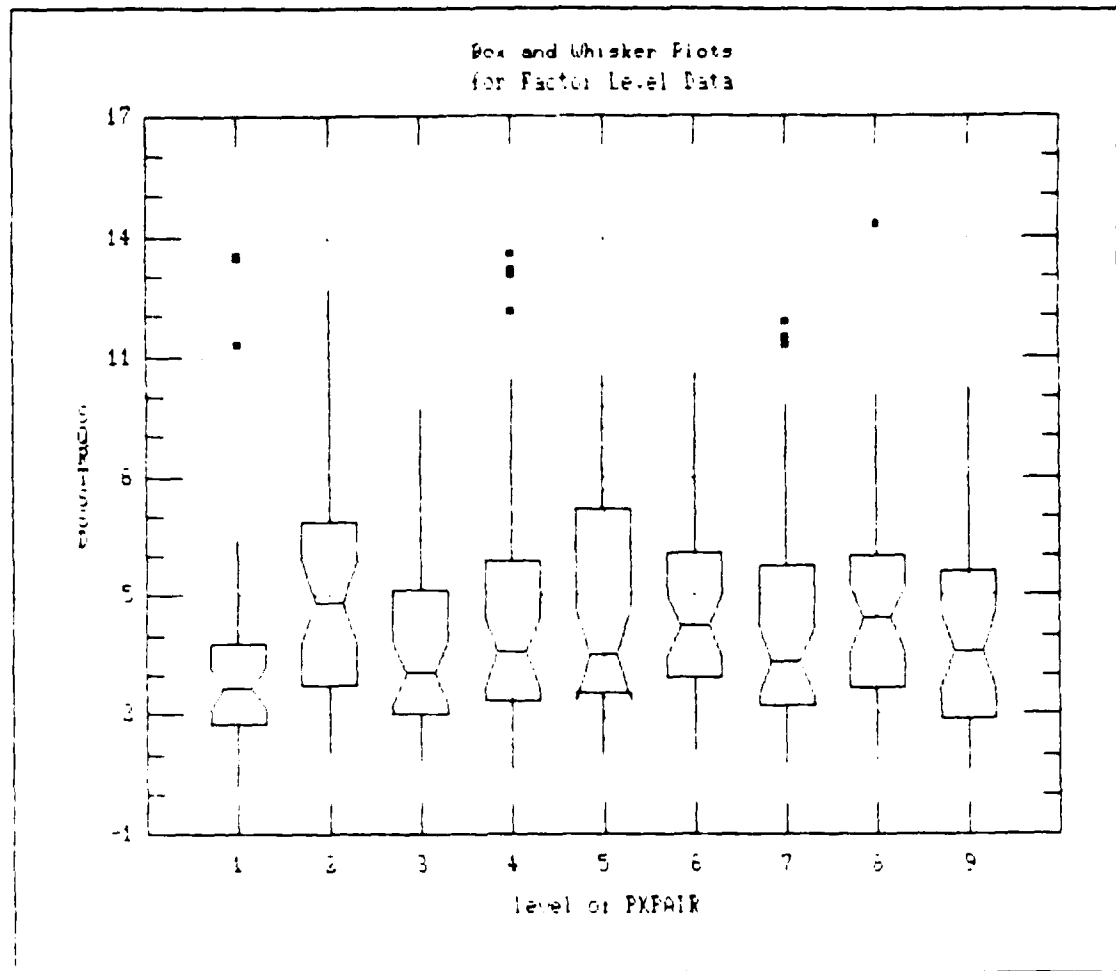


Figure H.2 Notched Box Plots of SqrtSSD as a Function of PKPAIR.

Figure H.2 displays the notched box plots of the square root of the sum of squared differences (SqrtSSD) as a function of probability of kill pairs (PKPAIRs). There are forty counts (ten counts for each of the four target selection methods) represented by each notched box. The "waist" indicates the mean and the length of the box is the middle fifty percent.

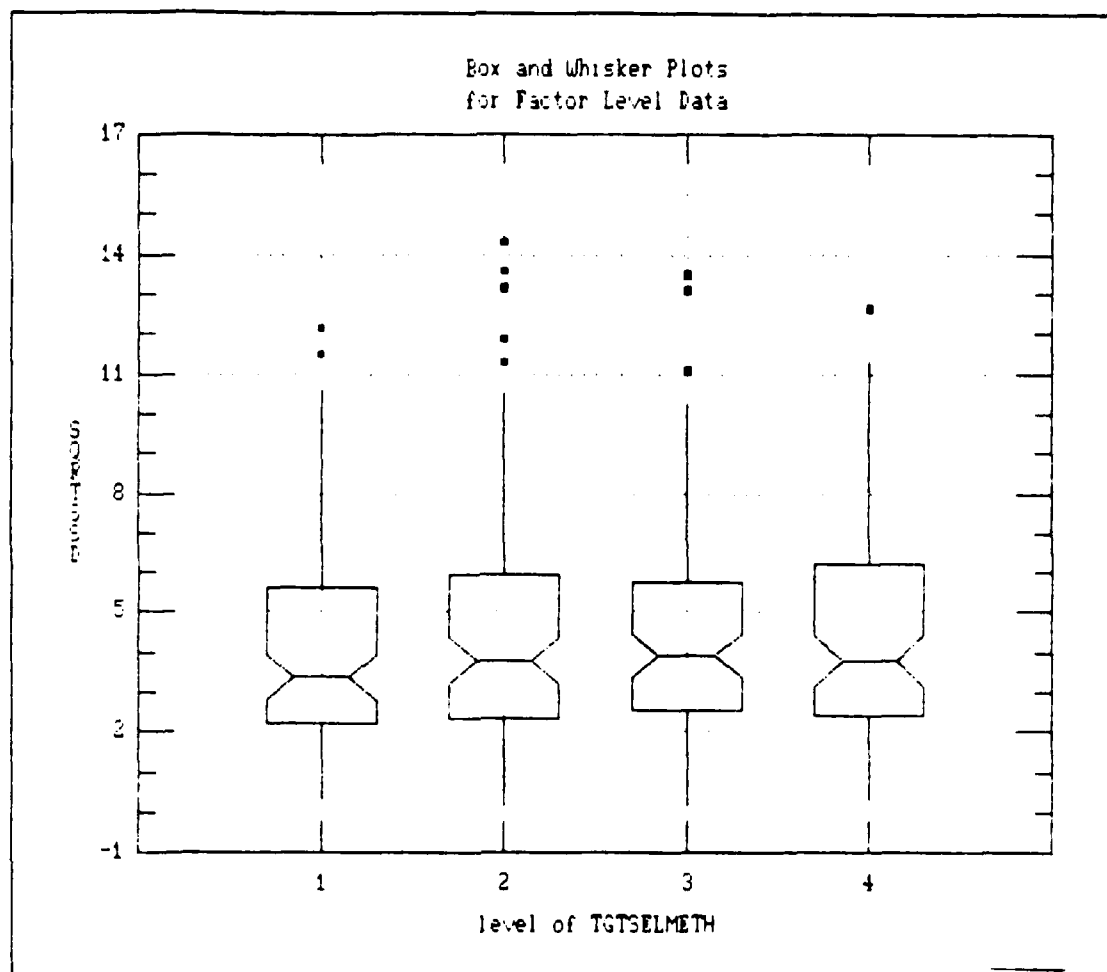


Figure H.3 Notched Box Plots of SqrtSSD as a Function of TGTSELMETH.

Figure H.3 displays the notched box plots of the square root of the sum of squared differences (SqrtSSD) as a function of target selection method (TGTSELMETH). There are ninety counts (ten counts for each of the nine probability of kill pairs) represented by each notched box. The "waist" indicates the mean and the length of the box is the middle fifty percent.

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